Environmental Activism: Endogenous Risk and Asset Prices*

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Abstract

We examine the relative effectiveness of three environmental activist strategies: Exit (divestment of shares), Boycott (of goods), and Voice (proxy-voting). We consider a four-period economy with two firms, green and brown, where the brown firm generates pollution (e.g. carbon emissions). A costly abatement technology is available, though without activism, the brown firm's manager, who maximizes shareholder value, will not abate even though it is socially optimal to do so. Boycott is always at least as effective as Exit and Voice is most effective, requiring the fewest activists to implement provided the brown firm is small. We find that successful divestment can create a time series in which green shares outperform brown shares, but then crash, underperforming brown shares. Over the long run in this setting, if activism is successful, brown shares underperform green shares in the long run. If activism does not succeed, activists have paid a high price for green shares and bear the cost of activism going forward.

JEL classification codes: D62, G12, L21

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1 Introduction

A burgeoning literature considers the role of investors and consumers in altering firm behavior towards reducing carbon emissions.[1] In this paper we examine the efficacy of, and returns generated by, the canonical and (by assumption) mutually exclusive strategies of divestment, engagement (e.g. activist voting), and consumer boycotts. We examine the effects of these strategies on share prices and managerial behavior in a four-period model. There are two firms, brown and green, and a costly abatement technology that can convert the brown production process to green. Utility-maximizing agents in period 0 are unaware of environmental issues. In period 1, agents become aware of future pollution from the brown firm. In periods 1 and 2, subsets of investors incrementally become environmental activists who jointly adopt one of the three strategies. We assume this concern about the environment is exogenous; specifically, we do not explicitly model social concerns as in Broccardo et al. (2022) and we assume that activists do not collaborate.[2]

With any of the strategies, there are levels of activism such that abatement results. We characterize these levels and discuss the relative likelihoods of success. We assume that if a strategy is not successful, the government taxes emissions, but at a level too low to by itself induce adoption of the green technology.

In expositing the model, we focus on divestment—selling brown shares and buying green shares—as this has been the strategy discussed most prominently over the last decade. A sufficient increase in the number of activist investors raises the price of the green shares and lowers the price of the brown shares from which the activists have divested.[3] We show that there is a critical threshold for activist shareholders, above which the divestment is successful and the brown firm adopts the costly green technology. Adoption occurs when the manager assesses the price of the firm with and without adoption and concludes that adoption maximizes the share price. At the same time, however, adoption of the green technology triggers a crash in the green share price as investors become willing to hold both

[1]Papers in this literature closely-related to ours include Heinkel et al. (2001), Pástor et al. (2021), Broccardo et al. (2022), Berk and van Binsbergen (2022), and Edmans et al. (2022).

[2]As documented in Dimson et al. (2015), collaboration among activists increases effectiveness.

[3]In our framework, when there are enough activist investors, Exit and Boycott will increase the expected return on the brown firm and depress its share price, consistent with the findings in Hong and Kacperczyk (2009), Chava (2014), Pástor et al. (2022), and Bolton and Kacperczyk (2021a).

green and brown shares. The brown share by contrast does not rally, but remains at its reduced price as it bears the cost of the technology. Successful divestment thus can create a time series in which green shares outperform brown shares, but then crash, underperforming brown shares. Over the long run in this setting, if activism is successful, brown shares underperform green shares in the long run. If activism does not succeed, activists have paid a high price for green shares and bear the cost of activism going forward. We believe the specific time series we predict are novel.

In general we show that Boycott is more effective than Exit in the model, in that the thresholds are no greater in a Boycott regime. Depending on the relative size of brown and green firms, Voice can be more or less effective than Exit or Boycott. In general, activists pay a price for their activism: under Exit, when they sell brown shares they do so at a low price; under Boycott, they pay more for green goods; and under Voice, when they accumulate brown shares they may have to pay a high price.[4]

We study the different strategies and characterize the price effects in a setting where the mechanism is transparent. There are important caveats to all of the strategies, however. Exit and Voice are applicable only to public companies. Boycott, on the other hand, requires the ability to identify goods by their origin, but it is applicable to both public and private companies. Taxation is effective but is generally believed to be politically infeasible. It is important to keep in mind that taxation and Boycott will be applicable to all firms irrespective of whether they are public or private companies, whereas Exit and Voice are applicable only for firms with publicly traded shares. However, we assume that the emissions tax is low and hence the green technology will not be adopted without the intervention of activists.[5]

As previously mentioned, there is a large literature in this area. Related to the approach in our paper, Heinkel et al. (2001) is a seminal contribution, studying in a one-period setting the comparative statics associated with socially responsible investors tilting their portfolios towards green firms. They perform calibrations to assess the percentage of green investors

[4]Hwang et al. (2021) document that firms after revelation of higher SRI ownership have negative stock returns and firm values come down after anticipated increase in CSR activities.

[5]Golosov et al. (2014) develop a general equilibrium model and find that the optimal tax on fossil fuel should be higher than the median estimates in the literature. Nordhaus (2019) argues that the price of CO_2 is much lower than optimal.

necessary to flip a brown firm to green. Pástor et al. (2021) generalize the framework in Heinkel et al. (2001), showing that a preference by some investors for green stocks gives rise to a three-factor pricing model including a green factor, and that green firms will have a lower cost of capital and therefore invest more. As a counterpoint to the general idea that tilting green is beneficial, Edmans et al. (2022) study a model where tilting towards brown firms can be a reward to the firm for making marginal green investments. Berk and van Binsbergen (2022) argue that as an empirical matter, divesting is likely to have little effect on the cost of capital of a brown firm, and that engagement is therefore likelier to be productive. Hartzmark and Shue (2023) point out that tilting towards green firms will generate only small environmental benefits since the firms are already green, and they further argue that the concomitant increase in the cost of capital for brown firms will discourage new green investments.[6]

Broccardo et al. (2022) take a different approach, allowing activists to have prosocial motives, taking into account the welfare of other investors. They show that the effectiveness of the two strategies depends on the intensity of prosocial sentiment. Also considering social responsibility, Oehmke and Opp (2020) study how prosocial investors can effect change by maximizing social value, rather than private value. In contrast, we do not model the microfoundations of activism.

How to achieve the socially responsible activities by private firms has been a widely discussed issue in the literature. Besley and Ghatak (2017) consider three types of organizations, social enterprise, non-profits, and for-profits, and the important role of citizen-managers with non-selfish preferences in running firms with flexible missions. Chowdhry et al. (2019) examine a project that produces profits and a social good at the same time, with two types of investors, one motivated by profit and another by social impact.[7] They show that when the

[6]Other papers study the effects of activism on firms. Albuquerque et al. (2019) build a model where customers are more loyal to green products, so firm has an incentive to go green. Akey and Appel (2019) empirically show that divestment leads to reduction in emissions due to brown output coming down. Naaraayanan et al. (2021) find that the drop comes from emission reducing technology adoption and not from reduction in output. Bolton and Kacperczyk (2021b) empirically find that greener firms have lower cost of capital, due to institutional investors divesting from brown shares. Gantchev et al. (2021) provide the evidence that a sufficiently large number of investors, even though they do not have large stakes in the firm, can affect the share price through their E&S preferences and induce the firm to improve E&S policies.

[7]Bansal et al. (2022) argue that investors' concern about socially responsibility is higher during good times. Riedl and Smeets (2017) and Barber et al. (2021) provide the evidence of investors' social preferences.

project's social impact is large, joint financing by both types of investors can be mutually beneficial. In our model, too, we consider a case of two types of investors, one who care only about profits and another who care about social goods as well, but our interest is in examining the effectiveness of various strategies available to those who care about social goods.

Various approaches to socially responsible investors' behavior have been explored in the literature. Gollier and Pouget (2014) examine a model where some investors are socially responsible and take externalities into account, but their focus is on large investors. They show that a large activist investor can profit by buying out a non-socially responsible firm and selling it after converting it into a responsible one.[8] Goldstein et al. (2022) also consider two types of investors — ESG and non-ESG. However, they focus on the informational content of asset price from which outside investors can learn the monetary prospects of a firm. By contrast, we exploit the incentivizing aspects of asset prices.

Activism by socially conscious investors can lead to legislative action, like taxation of brown outputs, and convince other agents in the economy to become activists. Our model does not directly reflect this aspect of reality but leave such features for future research.[9]

Finally, there are survey articles, including Kitzmueller and Shimshack (2012), who synthesize the literature on corporate social responsibility and explore why it exists. Besley and Ghatak (2018) review the literature on the role of incentives in providing goods and services that have returns with significant social components. Matos (2020) surveys the literature from the perspective of industry practitioners. Christensen et al. (2021) review the literature on economic effects of mandated disclosure for corporate social responsibility and sustainability. Our paper contributes to this broader literature on corporate social responsibility, emphasizing environmental responsibility and focusing on implementation mechanisms.

The rest of the paper is organized as follows. Section 2 lays out the structure and underlying assumptions of the economy in our model. Section 3 gives a detailed description

Starks et al. (2020) find that the use of ESG considerations are influencing investment decisions of investors with longer investment horizons – in both US mutual funds as well as institutional investors, which means that investors' sensitivity to ESG issues while making portfolio choice decisions is here to stay.

[8] Pedersen et al. (2021) develop a four-factor equilibrium asset pricing model when an asset's ESG score conveys information about the firm's fundamentals in addition to its contribution to negative social externalities.

[9] See Dunn et al. (2018), Jagannathan et al. (2018), Hsu et al. (2020), and Ardia et al. (2020).

of the timeline of the model and presents the decision problems of agents and firms. Section 4 defines and solves the equilibrium under each activist strategy. Section 5 examines heterogeneous firm sizes and endowments. Section 6 summarizes the model and concludes.

2 Structure of the Economy

We consider a stylized four-period economy ($t = 0, 1, 2, 3$) with a continuum of agents. Each agent is endowed with the non-storable consumption good in periods 0, 1 and 2 and in period 0 also endowed with shares in two firms, green and brown, that pay consumable dividends in period 3, the only period in which they operate. Production by the brown firm creates an externality that reduces the utility of all agents; a costly abatement technology, which we call the "green technology", is available. The only uncertainty in the model concerns the fraction of agents who become activists. For analytical convenience, we abstract away from other risk considerations that affect portfolio choice and stock prices.[10] Finally, if the green technology is not adopted, a tax is imposed at rate τ on consumption of goods produced by the brown firm when when the green technology is not adopted. Tax proceeds are rebated equally across all agents.

To briefly summarize the timeline:

- In period 0, agents receive initial share allocations and trade. Agents are unaware of pollution and activism in the future.
- In period 1, some agents are aware of future pollution and become environmental activists. All others are non-activists. They trade shares and the brown firm manager decides whether or not to adopt the technology.
- In period 2, a random number of non-activists become additional activists. Agents trade and the brown firm manager reconsiders the technology adoption decision (if not adopted at $t = 1$).
- In period 3, agents consume output from the brown and green firms.

[10]When there is economy-wide pervasive risk, there will be an additional utility cost to holding concentrated positions by deviating from the market portfolio. This should not directionally affect our main results.

Actions taken by the agents have the opportunity to influence the technology adoption decision. The central question is whether the green technology is adopted by the brown firm, and what may happen to the share prices when there is a sudden increase in the number of activists.

2.1 Agents

Agents live 4 periods and have CARA utility with coefficient A and without time discounting.[11] Consumption by agent i in period t is c_{it}. Per capita consumption of the public bad produced by the brown firm (if it does not adopt the green technology) is b, and agents are unaware of b at $t = 0$. The lifetime utility of agent i at time t is therefore[12]

$$U_{it} = \sum_{v=t}^{3} -e^{-A\left(c_{iv} - b\mathbf{1}_{v=3,t\neq 0}\right)}, \tag{2.1}$$

where $\mathbf{1}$ is an indicator function.

Each agent i at birth is endowed with

- θ_{ij} shares of each firm $j \in \{B, G\}$ at $t = 0$.
- ψ_{it} of the consumption good, received at the beginning of periods $t = 0, 1, 2$. We use ψ_{it} to denote consumable endowments in periods $0, 1$, and 2, and D_{B3} and D_{G3} to denote the consumable dividend paid by the brown and green firms in period 3, in which there is no endowed consumption.
- A type, either activist $\mathcal{A}$ (fraction k_t of agents, $i \in [0, k_t], t \in \{1, 2\}$), or non-activist $\mathcal{N}$ (fraction $1 - k_t$ of agents, $i \in (k_t, 1], t \in \{1, 2\}$). Activists wish to reduce the negative externality even if the action incurs a personal cost.

At $t = 2$, there is an exogenous probability q such that the fraction of activists jumps from k_1 to $k_2 = k_1 + \Delta$, where $\Delta > 0$ if the technology was not adopted at $t = 1$.[13]

[11] The main results for our paper are qualitatively identical under CRRA utilities.

[12] In our model, the public bad enters additively in the utility function as in Pástor et al. (2021).

[13] At $t = 1$ every agent knows about this possibility, but non-activists at that time do not believe that they will become activists.

Adoption of the green technology reduces output. We assume that the cost of adoption, ηD_{B3}, exceeds the consumption equivalent of the public bad, δD_{B3}:

$$\delta > \eta. \tag{2.2}$$

If the brown firm's manager does not adopt the green technology, consumption goods produced by the brown and green firms are distinct and denoted as c_{iB3} and c_{iG3}. Apart from their origins, the two goods are perfect substitutes, so that agent i's total consumption in period 3 is $c_{i3} = c_{iB3} + c_{iG3}$. Consumption goods received as endowments in earlier periods have no labels and are interchangeable.

When activists and non-activists have different per capita endowments, we write aggregate consumption in periods 1 and 2 as

$$\psi_t = k_t \psi_t^{\mathcal{A}} + (1 - k_t)\psi_t^{\mathcal{N}}; \qquad t \in \{1, 2\} \tag{2.3}$$

In the base case, with homogeneous endowment across all agents, we set $\psi_t^{\mathcal{A}} = \psi_t^{\mathcal{N}}$, and we simply denote ψ_t as the aggregate consumption at $t = 0, 1$, and 2.

Individual consumption and endowments of shares and goods are expressed in terms of *intensity* for an infinitesimal agent i. Aggregate consumption, for example, is $\int_0^1 c_i di = c$, where we use the same notation, c, for individual consumption intensity and aggregate consumption.

2.2 Firms

There are two firms, brown (B) and green (G), each of which has one share outstanding. Firms produce outputs, D_{B3} and D_{G3}, only in period 3. Outputs are converted into consumption goods, which are paid out to the shareholders of the respective firms as liquidating dividends, D_{B3} and D_{G3}. Consumption goods received as dividends are tagged so that agents can identify whether they are from firm B or G. The green firm always converts its output one-for-one into D_{G3} units of consumption good. Output of the brown firm, by contrast, depends on adoption of the green technology determined in period 1 or 2. There are two possibilities:

- **The green technology is not adopted.** In this case, the brown firm converts D_{B3} units of output into D_{B3} units of consumption good. Firm B produces b units of public bad as well, which adversely affects all agents in the economy equally. The scale factor that converts public bad into its consumption equivalent is δ, so that $b = \delta D_{B3}$, which is not a choice variable by agents. The output is tagged, so strategy such as Boycott are feasible. Goods produced by the brown firm are taxed at rate τ.

- **The green technology is adopted.** If the green technology is adopted by firm B, no public bad will be produced when firm B converts its output to consumption good, but conversion is less efficient: each unit of output will be converted to $1 - \eta$ units of consumption good, where $0 < \eta < 1$. There is no tax on goods produced by firm B.

2.3 Markets

Stocks are traded during periods $0, 1,$ and 2. In periods 1 and 2, trading takes place after agents learn their type ($\mathcal{A}$ or $\mathcal{N}$) and (in period 2), after the percentage of new activists, Δ, is resolved. In period 3, following the technology adoption decision, liquidating dividends are paid to shareholders, who then trade and consume the dividend. The consumption good is the numeraire in periods $0, 1,$ and 2, and the consumption good produced by firm G is the numeraire in period 3.

2.4 Activist strategies and behaviors

Activists make decisions without regard for personal cost. We make the following behavioral assumptions:

- **Exit strategy:** Activists will liquidate all the shares of firm B in period 1 and will not hold those shares at $t = 2$ if the green technology is not adopted.

- **Boycott strategy:** Activists will avoid the consumption good produced by firm B in period 3 if the green technology is not adopted.[14]

[14]The Boycott strategy induces a lexicographic preference for the green consumption good. Similarly, the

- **Voice strategy:** Activists will liquidate their holdings of green shares at $t = 1$ and 2 and invest the proceeds in shares of firm B in order to participate in a proxy vote in period 2 that replaces the non-activist manager with an activist manager, who will adopt green technology.

In any event, we assume that the government levies a tax at rate τ on the brown consumption good paid out as dividends if the green technology was not adopted. How the activists will behave is public knowledge. There is only one source of uncertainty in this economy — whether some of the existing non-activists become activists in period 2.

3 Decision Problem of Individual Agents and Firms

We now describe the consumption-portfolio choice problem of each type of agent. Non-activists choose consumption and share holdings to maximize utility taking the externality as given. They value shares of brown and green firms based on their dividends and treat the consumption goods produced by the two firms as perfect substitutes. Activists also maximize utility, but are subject to the behavioral constraints outlined in Section 2.4. Throughout, we assume no short-selling of shares.

3.1 Timeline

Figure 1 summarizes the timeline of the model, and in particular Figure 2 depicts the evolution of events and the decisions that the agents and brown firm's manager make at $t = 1$ and $t = 2$.

Period 0. Each agent is endowed with θ_{iB} of firm B's shares, θ_{iG} of firm G's shares, and ψ_{i0} of consumption good. Given share prices p^s_{B0} and p^s_{G0}, agents choose consumption c_{i0} and shareholdings θ_{iB0} and θ_{iG0}.[15] At this stage, all agents are unaware of pollution and believe there will be no activist in the future.

Period 1. A fraction k_1 of agents are now aware of environmental issues and become

Exit strategy is as if the agents had lexicographic preference for the green firm's shares. By contrast, in Pástor et al. (2021) an agent's preference for a firm's characteristics is continuous.

[15] Shareholdings of θ_{ij1} and θ_{ij2} are the holding *intensities* chosen by an infinitesimal agent i.

activists. All agents choose consumption c_{i1} and shareholdings θ_{iB1} and θ_{iG1} given share prices p^s_{B1} and p^s_{G1}. In the case of Exit, activists will divest all their holdings of the brown firm's shares if the technology is not adopted. Under Exit and Boycott, we assume that the non-activist manager adopts the green technology only if doing so leads to a higher share price at $t = 1$. Under Voice, activists hold only brown shares in order to become the majority shareholders of the brown firm, and there is no voting nor adoption at $t = 1$.

Period 2. Each agent i enters period $t = 2$ holding θ_{iB1} shares of firm B, θ_{iG1} shares of firm G, and an endowment of consumption good of ψ_{i2}.[16] If the manager adopted the green technology at $t = 1$, the fraction of activists at $t = 2$ is irrelevant. If the manager did not, the fraction of activists at the beginning of $t = 2$ is determined by nature:

- With probability $1 - q$, there is no change in the composition of agents (state S_{21}).
- With probability q, some of the existing non-activists become activists, and the fraction of activists in the economy jumps to $k_2 = k_1 + \Delta$ (state S_{22}). By default, firm B's manager is non-activist.
- Under Exit and Boycott, the non-activist manager adopts the green technology, if not adopted at $t = 1$, when doing so yields a higher share price at the beginning of period 2. Otherwise, the firm remains brown.
- Under Voice, the manager type is determined by an election at $t = 2$ after realization of k_2, in which a majority vote determines whether there will be a switch to an activist manager. If the vote is successful, the brown firm will adopt the green technology.
- In each of the states $S_{1N}, S_{1A}, S_{21N}, S_{22N}, S_{22A}$, and S_{23A}, agents choose consumption c_{i2} and shareholdings θ_{iB2} and θ_{iG2} given share prices p^s_{B2} and p^s_{G2} in that state.

Period 3. At $t = 3$, there are four possible states denoted by $S_{31N}, S_{32N}, S_{32A}$, and S_{33A}, and they are subsequent states of $S_{21N}, S_{22N}, S_{22A}$, and S_{23A}, respectively. Each agent i enters period 3 holding θ_{iB2} shares of firm B and θ_{iG2} shares of firm G, possibly different for different states, following which the outputs of firms B and G are realized and converted into final consumption goods that are paid out as dividends.

[16] In the Exit equilibrium, $\theta_{iB1} = 0$ for each activist.

Firm G pays a dividend of D_{G3}. Firm B's dividend, D_{B3s}, depends on adoption of the green technology. Without adoption, the dividend is $D_{B3} = D_{B31N} = D_{B32N}$, and firm B generates δD_{B3} units of public bad and the government taxes firm B's consumption good at the rate τ. The government redistributes the tax revenue uniformly to all agents. If instead the green technology is adopted, firm B will produce $(1-\eta)\, D_{B3}$ units of consumption goods and there is no tax. In each of the states, agents choose consumption c_{iB3} and c_{iG3} given the price of brown consumption goods.

3.2 Optimization problem of individual agents

We now describe the optimization problems of individual agents in each period. Recall that an activist is indexed by $i \in [0, k_t]$ and a non-activists is indexed by $i \in (k_t, 1]$. In period t, share prices of firms B and G are p^s_{Bt} and p^s_{Gt}.

Period 0. The numeraire is the consumption good at $t = 0, 1$ and 2 and good produced by firm G at $t = 3$. Agents have no knowledge of pollution and future activism. Each agent $i \in [0, 1]$ takes prices as given and solves the following problem

$$U_{i0} = \max_{\theta_{iB0}, \theta_{iG0}, c_{i0}} \left\{ -e^{-Ac_{i0}} - e^{-Ac_{i1}} - e^{-Ac_{i2}} - e^{-Ac_{i3}} \right\}, \tag{3.1}$$

subject to the budget constraint $c_{i0} + \theta_{iB0} p^s_{B0} + \theta_{iG0} p^s_{G0} = \theta_{iB} p^s_{B0} + \theta_{iG} p^s_{G0} + \psi_{i0}$.

Period 1. Agents are now aware of pollution at $t = 3$. The numeraire in this period is the consumption good. Under the Voice and Boycott strategies, both activists and non-activists face the same problem at $t = 1$. Each agent $i \in [0, 1]$ takes prices as given and decides how much to consume and what portfolio to hold by maximizing

$$U_{i1} = \max_{\theta_{iB1}, \theta_{iG1}, c_{i1}} \left\{ -e^{-Ac_{i1}} + \mathbf{E}_1 \left[U_{i2} \left(\theta_{iB1}, \theta_{iG1} \right) \right] \right\}, \tag{3.2}$$

subject to the budget constraint $c_{i1} + \theta_{iB1} p^s_{B1} + \theta_{iG1} p^s_{G1} = \theta_{iB0} p^s_{B1} + \theta_{iG0} p^s_{G1} + \psi_{i1}$, where U_{i2}, to be specified below, is the period-2 utility of agent i who takes into account the public bad at $t = 3$. $\mathbf{E}_1$ is the expectation with respect to uncertainty in the activist population at $t = 2$ and the subsequent technology adoption rules to be specified in Section 3.3. Note that each non-activist does not think that they will become activists themselves.

Under the Exit strategy, activists are subject to the additional constraint that $\theta_{iB1} = 0$ for $i \in [0, k_1]$. Under Voice, activists will not hold green shares, i.e., $\theta_{iG1} = 0$ for $i \in [0, k_1]$.

Period 2. The numeraire in this period is again the consumption good. Under the Boycott strategy, both types of agents face the same problem at $t = 2$. In each state, each agent $i \in [0, 1]$ takes prices as given and decides how much to consume and what portfolio to hold by maximizing

$$U_{i2}\left(\theta_{iB1}, \theta_{iG1}\right) = \max_{\theta_{iB2}, \theta_{iG2}, c_{i2}} \left\{-e^{-Ac_{i2}} + U_{i3}\left(\theta_{iB2}, \theta_{iG2}\right)\right\}, \tag{3.3}$$

subject to the budget constraint $c_{i2} + \theta_{iB2} p^s_{B2} + \theta_{iG2} p^s_{G2} = \theta_{iB1} p^s_{B2} + \theta_{iG1} p^s_{G2} + \psi_{i2}$, where U_{i3} is the period-3 utility of agent i to be specified below.

Under Exit, activists continue to solve the above optimization problem, but they will be subject to an additional constraint: $\theta_{iB2} = 0$ for $i \in [0, k_2]$ in states S_{21N} and S_{22N}. Under Voice, activists will not hold green shares: $\theta_{iG2} = 0$ for $i \in [0, k_2]$.

Period 3. At $t = 3$, we use the consumption good produced by firm G as the numeraire. Denote p^c_{B3} as the pre-tax price of consumption good from firm B. Under the Exit and Voice strategies, both activists and non-activists face the same problem at $t = 3$. In each of the states, each agent i chooses how much of firm G's and firm B's consumption goods to consume so as to maximize the utility

$$U_{i3}\left(\theta_{iB2}, \theta_{iG2}\right) = \max_{c_{iB3}, c_{iG3}} \left\{-e^{-A\left(c_{iB3} + c_{iG3} - \delta D_{B3}\left(1 - \mathbf{1}_A\right)\right)}\right\} \tag{3.4}$$

subject to the budget constraint

$$c_{iG3} + p^c_{B3}\left(1 + \tau\left(1 - \mathbf{1}_A\right)\right) c_{iB3} = \theta_{iG2} D_{G3} + \theta_{iB2} p^c_{B3}\left(D_{B3} - \eta D_{B3} \mathbf{1}_A\right) + \tau p^c_{B3}\left(1 - \mathbf{1}_A\right) D_{B3},$$

where $\mathbf{1}_A$ denotes the indicator function that takes the value of 1 if the technology is adopted and 0 otherwise. The budget constraint includes terms reflecting the cost of adoption, η, as well as the tax and tax rebate.

Under the Exit and Voice strategies, the maximization problem in (3.4) is straightforward and can be reduced to maximizing the total quantity of consumption, $c_{iB3} + c_{iG3}$. Under Boycott, activists will be subject to an additional constraint: $c_{iB3} = 0$ for $i \in [0, k_2]$ in states S_{31N} and S_{32N}.

To summarize, all non-activists solve standard portfolio-choice problems at each point in time, taking into account the uncertainty in the population of activists at $t = 2$, while they do not think themselves will become activists. Each activist is subject to an additional constraint depending on which strategy we analyze.

3.3 Decision problem of firms

The manager of firm G makes no decisions. The manager of firm B, who is not an activist, can adopt the green technology at a cost of reducing output by the fraction η. Adoption is publicly observable. The objective of firm B's existing non-activist manager is to make the technology decision that maximizes the value of the shares at the time when the decision is made.[17] To make the decision under Exit and Boycott, the manager needs to compare firm B's value along two possible equilibrium paths, adoption and no adoption, adopting the green technology if and only if it results in a higher firm value.

At $t = 1$, firm B's manager will select state S_{1A} (adoption) or S_{1N} (no adoption) depending on which state will have a higher brown share price. If the technology is not adopted at $t = 1$, the manager will again make a decision at $t = 2$, if there is an increase in activist population. The adoption rule of the default non-activist manager depends on the stock price conditional on adoption:

$$\begin{cases} S_{1A} \Longleftrightarrow p^s_{B1}\left(S_{1A}\right) > p^s_{B1}\left(S_{1N}\right) \\ S_{22A} \Longleftrightarrow p^s_{B2}\left(S_{22A}\right) > p^s_{B2}\left(S_{22N}\right) \end{cases} \tag{3.5}$$

With an activist manager, by contrast, the technology is always adopted.

The voice strategy differs in that shareholders can change the manager, from the non-activist default to one who will adopt the green technology.

[17] We have assumed the firms are large enough so that manager's decision will affect share prices, but the manager will not deliberately manipulate prices. We leave the study of infinitesimal firms for future research.

4 Equilibrium

In this section, we define equilibrium and present numerical solutions of the model. We first discuss the benchmark economy with no activists and an emissions tax too small to induce adoption of the green technology. We then allow activists to follow either Exit, Boycott, or Voice. Throughout this section, we assume homogeneous endowments among each group of agents but allow heterogeneity between activists and non-activists, and we also allow the green and brown firms to be of different sizes.

In Section 4.1, we define the benchmark equilibrium, in which there is an emissions tax and activists do not engage in activist strategies. In Section 4.2, we examine the equilibrium when all agents have identical endowments of shares and consumption goods, while in Section 5.1 we examine the equilibrium when activists and non-activists differ in their endowments but are identical within their group. In Section 5.2, we allow heterogeneous firm sizes.

4.1 Definition of benchmark equilibrium

Definition 4.1. In the benchmark equilibrium:

- There are no activists[18]
- $\tau < \overline{\tau} \equiv \frac{e^{A(\delta-\eta)D_{B3}}}{1-\eta} - 1$ (firm B does not convert to green technology; Proposition 4.1, below)
- a set of the consumption and portfolio holdings given by $(c_{it}, \theta_{iBt}, \theta_{iGt})$ for each agent $i \in [0,1]$ in periods $t = 0, 1$, 2, and with period 3 consumption given by (c_{iB3}, c_{iG3})
- a technology adoption decision rule given in (3.5)
- prices of shares of firms B and G in periods 0, 1 and 2, and price of the consumption good produced by firm B in period 3 given by the price vector $(p^s_{B0}, p^s_{G0}, p^s_{B1}, p^s_{G1}, p^s_{B2}, p^s_{G2}, p^c_{B3})$

such that

[18] Therefore, the uncertainty of k at the beginning of $t = 2$ is irrelevant.

(i) given the price vector and the technology adoption rule, the consumption and portfolio holdings in each period solve the maximization problems given in equations 3.1–(3.4);

(ii) the markets for consumption goods and shares clear given the consumption and portfolio holdings at $t = 0, 1, 2$, consumption at $t = 3$, and the price vector. The market clearing conditions are given by: $\int_i c_{i0} di = \int_i \psi_{i0} di = \psi_0, \int_i c_{i1} di = \int_i \psi_{i1} di = \psi_1, \int_i c_{i2} di = \int_i \psi_{i2} di = \psi_2, \int_i c_{iG3} di = D_{G3}$ and $\int_i c_{iB3} di = D_{B3} - \eta D_{B3} \mathbf{1}_A$ for the consumption goods market, and $\int_i \theta_{iB0} di = \int_i \theta_{iG0} di = 1$, $\int_i \theta_{iB1} di = \int_i \theta_{iG1} di = 1$ and $\int_i \theta_{iB2} di = \int_i \theta_{iG2} di = 1$ for the stock market.

Proposition 4.1. *In the Benchmark equilibrium, the green technology is not adopted by the brown firm's value-maximizing manager if $\tau < \overline{\tau}$, where*

$$\overline{\tau} = \frac{e^{A(\delta - \eta) D_{B3}}}{1 - \eta} - 1. \tag{4.1}$$

Most of our derivations will be in Appendix A, but we describe here the proof of Proposition 4.1 to illustrate the workings of the model. The question is whether the manager of the brown firm will adopt the green technology. This decision has an effect only in period 3, in which production occurs and the tax is levied on output if the technology is not adopted. The manager decides by comparing the current share prices under non-adoption and adoption and takes the action that maximizes the share price.[19] Standard calculations, detailed in Appendix A, show that the discount factors at $t = 2$ are

$$M_{\texttt{AD}} \left(c_{i2}, D_{G3} + D_{B3}(1 - \eta) \right) = e^{A c_{i2} - A(D_{G3} + D_{B3}(1 - \eta))} \tag{4.2}$$

$$M_{\texttt{NAD}} \left(c_{i2}, D_{G3} + D_{B3} \right) = e^{A c_{i2} - A(D_{G3} + D_{B3}(1 - \delta))}. \tag{4.3}$$

If there is adoption, period 2 consumption is unaffected but period 3 consumption is reduced by the fraction η. If there is no adoption, the aggregate consumption is reduced by the externality, δD_{B3}. Shares provide a claim to period 3 output. Agents value consumption goods from both firms the same, so when the technology is not adopted, the price of the

[19]Note that in making the decision, the manager is comparing two equilibria and affecting the discount factor. This is the reason that even if $\eta = 0$, $\overline{\tau} > 0$.

brown good must be reduced by the factor $1/(1+\tau)$, so that the post tax price equals that of the green good. The brown share price is the discount factor times the cash flow. Thus, the ratio of brown share prices when the technology is not adopted (N) and when it is adopted (A) at $t = 2$ is

$$\begin{aligned}
\frac{p^s_{B2}(N)}{p^s_{B2}(A)} &= \frac{M_{\mathtt{NAD}}\left(\psi_2, D_{G3} + D_{B3}\right) \frac{D_{B3}}{1+\tau}}{M_{\mathtt{AD}}\left(\psi_2, D_{G3} + D_{B3}(1-\eta)\right) D_{B3}(1-\eta)} \\
&= \frac{e^{A(\delta-\eta)D_{B3}}}{(1-\eta)(1+\tau)}.
\end{aligned}$$

This expression is less than 1 (tax-induced adoption is optimal) when $\tau > \overline{\tau}$, as defined in equation (4.1) in Proposition 4.1. Thus, Proposition 4.1 defines the tax rate, above which the brown firm adopts the green technology. Carbon taxes are frequently discussed but infrequently enacted, consistent with the observations of Golosov et al. (2014) and Nordhaus (2019) that tax rates in practice are often lower than optimal due to institutional restrictions. Thus, we will retain the tax in the Benchmark equilibrium but assuming that $\tau < \overline{\tau}$.

Finally, going forward we define the interest rate to be the return on the green share, which is a claim to the green consumption good at $t = 3$. Note that in the benchmark equilibrium all agents are identical and there are no activists, so in order for markets to clear the return on brown and green shares must be the same.

In the following, we see how the economy changes with activism undertaken by activists. Agents of a given type ($\mathcal{A}$ or $\mathcal{N}$) are identical and make the same consumption and investment decisions in equilibrium. Hence, instead the subscript of i, we use superscripts $\mathcal{A}$ and $\mathcal{N}$ to distinguish the types. For example, we use $c_1^{\mathcal{A}}$ for consumption of an activist at $t = 1$.

4.2 Equilibrium with activist agents

We illustrate equilibrium under each activist strategy with numerical examples using parameters defined in Table 1. We choose these parameters so that in the Benchmark equilibrium, the one-period net returns from $t = 1$ onward on both brown and green shares are zero. We set $\delta > \eta$ as in expression (2.2).

4.2.1 Equilibrium under Exit

In the Exit strategy, activists sell brown shares in period 1 if the green technology is not adopted (state S_{1N}) and will not purchase brown shares at $t = 2$ in states S_{21N} and S_{22N}. If the technology is adopted, they take no action. The fraction of activists at $t = 2$ is the only uncertainty, and the conditional strategy of the activist shareholders in period 2 is rationally anticipated in period 1.[20] Activists will hold no brown shares in state S_{1N}, S_{21N} or S_{22N}:

$$\begin{cases} \theta^{\mathcal{A}}_{B1}\left(S_{1N}\right) = 0 \\ \theta^{\mathcal{A}}_{B2}\left(S_{21N}\right) = 0 \\ \theta^{\mathcal{A}}_{B2}\left(S_{22N}\right) = 0 \end{cases} . \tag{4.4}$$

We are interested in the effect of the Exit strategy on both share and consumption good prices. The following proposition establishes two important thresholds on the fraction of activists when $q = 0$, i.e., $k_2 = k_1$. We exhibit the intuition for case of $q > 0$ using examples.

Proposition 4.2. *In an Exit equilibrium when $q = 0$, there are thresholds $\bar{k}_{Exit}$ and $\bar{\bar{k}}_{Exit}$ such that*

- *if $k_1 < \bar{k}_{Exit}$, share prices will be the same as in the Benchmark equilibrium, and the green technology will not be adopted.*
- *if $\bar{k}_{Exit} < k_1 \leq \bar{\bar{k}}_{Exit}$, shares prices will deviate from prices in the Benchmark equilibrium, and the green technology will not be adopted.*
- *if $k_1 > \bar{\bar{k}}_{Exit}$, the green technology will be adopted at $t = 1$.*

When following the Exit strategy, activists do not distinguish green and brown goods. Therefore, the cum-tax price of the brown good must equal that of the (untaxed) green good. Thus, we have $p^c_{B3}(S_{31A}) = p^c_{B3}(S_{32N}) = \frac{1}{1+\tau}$ and $p^c_{B3}(S_{32A}) = p^c_{B3}(S_{33A}) = 1$.

With Exit, activists in period 1 sell all their brown shares in exchange for green shares. Non-activists are the counterparty. The behavior of activists is mechanical, but non-activists are willing holders of both brown and green shares, so the shares must be priced accordingly.

[20] By assumption, activist strategies are only present when the green technology is not adopted, which means that activists do not have preferences for green in states S_{1A}, S_{22A}, and S_{23A}.

The Exit strategy can work only if the fraction of activists is large enough for divestment to affect the brown share price significantly. “Significant” in this case means that divestment must drive the brown share price so low that the share price gain from adopting the green technology outweighs the cost of adoption. From this verbal description, it is clear that there are potentially three regions: no price effect for low k_1; a price effect insufficient to induce adoption for intermediate k_1; and adoption for large k_1. Figure 3 illustrates the existence of the two thresholds when $q = 0$. We will now examine an example when $q > 0$ and explain how we determine the cutoffs $\bar{k}$ and $\bar{\bar{k}}$.[21]

Example 1. Using the parameters in Table 1, we characterize the Exit equilibrium.

Small k_1: $k_1 \leq \bar{k}$

We define $\bar{k}$ as the largest value of k_1 for which the economy is unaffected by activists selling all their brown shares at $t = 1$. For the values in Table 1, $\bar{k} = 0.525$. In this case, brown and green shares are priced so that non-activists are indifferent toward which share they hold, and the price of each share is the present value of dividends. Thus, the price of the green shares is $p^s_{G1} = 0.570$ (the value to be paid in period 3), while the price of the brown shares is $p^s_{B1} = \frac{p^s_{G1}}{1+\tau} = 0.518$. Below $\bar{k}$, prices of both the brown and green shares are constant with respect to k_1. These results are plotted in Figure 4. We also report the numerical values in Table 2.

To understand how $\bar{k}$ is determined, let us consider the case of $q = 0$ as in Figure 3. Since all agents have equal endowments of green and brown shares at $t = 0$, and the two shares must have the same return when activists willingly hold brown and green shares. We may thus assume that all agents arrive at $t = 1$ holding their endowed shares. When the stock market opens at the beginning of $t = 1$, activists will divest brown shares. Therefore, non-activists in aggregate will exchange $1 - \bar{k}$ green shares for $\bar{k}$ brown shares from the activists. For non-activists to make this trade, the value of brown shares divested by activists must be equal to the value of green shares sold by non-activists. That is, $\bar{k} p^s_{B1} = (1 - \bar{k}) p^s_{G1}$. Solving, we obtain $\bar{k} = 0.524$.

This (and other results) obviously rely on the absence of cash flow risks. By way of

[21] We drop the subscript in the thresholds when unambiguous.

comparison, Heinkel et al. (2001) have cash flow risks, so even with few activist investors, prices are affected by divestment. However, if the cash flows from brown and green firms are highly correlated, then the risk is analogous to a systematic risk, and we would also expect no price effect in their model when the number of activists is sufficiently small.

Intermediate k_1: $\bar{k} < k_1 < \bar{\bar{k}}$

In this region, the aggregate wealth of activists is sufficient to buy all green shares. As a result, if k_1 is greater, the price of green shares is higher (more agents purchase the existing stock of green shares) and the price of brown shares (purchased by fewer agents) is lower. In this region, however, the brown price is not low enough for the price benefit of adoption (more agents willing to hold firm B's shares) to overcome the cost of adoption (reduced output). The effect of k_1 on share prices is apparent in Figure 4.

Large k_1: $k_1 \geq \bar{\bar{k}}$

In this region the Exit strategy is successful: the brown firm adopts the green technology. Output declines from D_{B3} to $(1-\eta)D_{B3}$, but the emissions tax is not levied. Figure 4 demonstrates the evolution of share prices for a range of k_1, and Table 3 provides time series of share prices and returns when $k_1 = 0.53$ (slightly above $\bar{k}$) and $k_1 = 0.57$ (slightly below $\bar{\bar{k}}$).[22] When there is no sudden increase in activist population at $t = 2$, the green share has a higher return than the brown share relative to $t = 0$ when there is no concern about the environment. This observation is consistent with Pástor et al. (2022) such that an increase in environmental concerns leads to outperformance of green shares. This trend continues when k_1 gets larger as long as $k_1 < \bar{\bar{k}}$.

More importantly, we also highlight the response of share prices to an unexpected increase in activists. Suppose the fraction of activists changes from $k_1 = \bar{\bar{k}} - \varepsilon$ to $k_2 > \bar{\bar{k}}$ at the beginning of $t = 2$, i.e., state S_{22} is realized. The economy then transitions from no adoption at $t = 1$ to adoption at $t = 2$. From Panel B of Table 3, we observe that the green share price at $t = 2$ falls significantly relative to period-1 price while the brown share price remains

[22] Note that the time period in calendar years from $t = 0$ to 1, 1 to 2, and 2 to 3 will not necessarily be the same.

almost unchanged: the green share has a realized return of about -9.7% while the brown share only decreases by around 0.8%. This is an example where the technology is adopted but green shares are riskier than brown shares.

Furthermore, from $t = 2$ to 3, if the green share price crashed in period 2, both shares will have the same return (state S_{32A}), albeit the brown firm producing a lower output; but if the green share did not crash, the brown share will have a much higher return (state S_{31N}). This implies that the activists will bear a loss either at $t = 2$ when the technology is adopted and high green share price cannot sustain, or at $t = 3$ when the technology was not adopted and the output is realized and equal to the brown output.

The threshold $\bar{\bar{k}}$ is the smallest k_1 at which the technology is adopted. The calculation of $\bar{\bar{k}}$ is complicated because it takes into account optimizing decisions in all periods. To understand the calculation, agents know in period 1 if $k_1 > \bar{\bar{k}}$. If so, the technology will be adopted. Uncertainty about the fraction of activists at $t = 2$ creates risk that affects agents' period-1 shareholdings. Once in period 2, whether the fraction of activists increases affects wealth and thus affects the realized marginal rate of substitution between periods 2 and 3; this in turn affects the valuation of shares in period 2. Because of the complexity of this calculation, we solve $\bar{\bar{k}}$ numerically. We use this numerical example to illustrate the existence of $\bar{\bar{k}}$. The proof of the existence and uniqueness of $\bar{\bar{k}}$ when $q = 0$ can be found in Appendix A.1. Note that when $k_1 \geq \bar{\bar{k}}$ the share price associated with non-adoption will be off-equilibrium and that price will never be observed by agents.

Figure 5 shows the utilities of both types of agents at $t = 1$. When $k_1 \leq \bar{k}$, the utilities are the same across all agents because the equilibrium is identical to the Benchmark equilibrium. When $\bar{k} < k_1 \leq \bar{\bar{k}}$, the activists' preferences for green will incur a utility cost as they buy more expensive green shares and sell cheaper brown shares. Non-activists benefit from this behavior.[23] Since non-activists are indifferent, they can purchase cheaper brown shares and have greater utility. Once the green technology is adopted ($k_1 > \bar{\bar{k}}$), firm B is essentially green. The equilibrium becomes the Benchmark equilibrium with lower firm B's output and no public bad. As a result, utilities of both types of agents are identical.

[23]In equilibrium, non-activists cannot have lower utilities than activists because they can always mimic the behaviors of activists.

4.2.2 Equilibrium under Boycott

Next, we consider the Boycott strategy, which is a boycott of brown firm consumption goods in state S_{31N} or S_{32N} when the technology is not adopted:

$$\begin{cases} c^{\mathcal{A}}_{B3}\left(S_{31N}\right) = 0 \\ c^{\mathcal{A}}_{B3}\left(S_{32N}\right) = 0 \end{cases}. \tag{4.5}$$

In our setting, the Boycott equilibrium resembles the Exit equilibrium because shares are a claim to period 3 consumption. The following proposition establishes two important thresholds on the fraction of activists when $q = 0$, i.e., $k_2 = k_1$. We exhibit the intuition for cases when $q > 0$ using examples.

Proposition 4.3. *In a Boycott equilibrium when $q = 0$, there are thresholds $\bar{k}_{Boycott}$ and $\bar{\bar{k}}_{Boycott}$ such that*

- *if $k_1 < \bar{k}_{Boycott}$, share prices will be the same as in the Benchmark equilibrium, and the green technology will not be adopted.*
- *if $\bar{k}_{Boycott} < k_1 \leq \bar{\bar{k}}_{Boycott}$, shares prices will deviate from prices in the Benchmark equilibrium, and the green technology will not be adopted.*
- *if $k_1 > \bar{\bar{k}}_{Boycott}$, the green technology will be adopted at $t = 1$.*

To find out the value of $\bar{k}$ for Boycott, we apply a similar argument as in the Exit case when $q = 0$. Suppose k_1 approaches $\bar{k}$ from the left. Activists have no preference toward green shares under Boycott, and the returns on green and brown shares must be equal. Since agents have homogeneous endowments, we may then assume agents arrive at $t = 3$ holding their endowed shares. Therefore, activists in aggregate will exchange $\bar{k}D_{B3}$ units of brown goods for $\bar{k}p^c_{B3}(S_{31N})D_{B3}$ units of green goods. In addition, they will use the tax rebate $\bar{k}\tau p^c_{B3}(S_{31N})D_{B3}$ to purchase green goods, so the total demand of green goods is $\bar{k}(1 + \tau)p^c_{B3}(S_{31N})D_{B3}$, where $p^c_{B3}(S_{31N}) = \frac{1}{1+\tau}$ as mentioned in Section 4.1. The non-activists will supply $(1 - \bar{k})D_{G3}$ units of green goods. Thus, we must have

$\bar{k}(1+\tau)p^c_{B3}(S_{31N})D_{B3} = (1-\bar{k})D_{G3}$. Since we set $D_{G3} = D_{B3} = 0.57$ and $\tau = 0.1$, we obtain that $\bar{k} = 0.5$.

The intuition for this proposition is similar to that for Proposition 4.2. It is natural to compare the adoption threshold levels of k_1 under the Exit and Boycott strategies when $q = 0$, which is given in the following proposition.

Proposition 4.4. *At $t = 2$ where the green technology is adopted under Exit equilibrium, it is also adopted under Boycott equilibrium. However, the converse does not hold.*

First, if there is no taxation, the two mechanisms are equivalent. In a Boycott equilibrium, if the brown firm manager does not adopt the green technology, activists will allocate all of their wealth to the green consumption goods. Since activists treat the two types of consumption goods differently, the price of brown consumption goods in units of green goods will not necessarily be 1 in state S_{31N} or S_{32N}. For the Boycott strategy to make a difference in the economy, there has to be a large enough difference in the prices of the green and brown goods. This requires that the activists consume all of the green good, which drives up its price. Similarly, under Exit, the activists allocate 100% of their savings to the green shares.

Exit and Boycott are equivalent when there are no taxes but differ when brown output is taxed. The tax widens the wedge between green and brown output prices, affects the share prices similarly, and enables boycott to succeed at a lower k_1. As a result, when $q = 0$, the period-3 consumption by a non-activist when there is a price effect ($k_1 > \bar{k}$) is $c_3^{\mathcal{N}}(exit) = \left(\frac{1}{1-k_1}\frac{1}{1+\tau} + \frac{\tau}{1+\tau}\right) D_{B3}$ under Exit and $c_3^{\mathcal{N}}(boycott) = \frac{1}{1-k_1} D_{B3}$ under Boycott. The first term in $c_3^{\mathcal{N}}(exit)$ is due to each non-activist holding of $\frac{1}{1-k_1}$ shares of the brown firm, and the second term is due to the uniform tax rebate. It is easy to see that $c_3^{\mathcal{N}}(exit) < c_3^{\mathcal{N}}(boycott)$, so we should expect a higher brown share price under Exit than under Boycott, leading to Boycott being a more effective strategy. Indeed, Lemma A.16 in the Internet Appendix shows such comparison.

Example 2. We assume the same set of parameters as in Example 1. The features and intuition of a Boycott equilibrium are similar to an Exit equilibrium discussed earlier except for one crucial difference as follows. We observe that both of the thresholds $\bar{k} = 0.5$ and $\bar{\bar{k}} = 0.554$ in the Boycott equilibrium are lower than their counterparts in the Exit equilibrium as mentioned in Proposition 4.4. This implies that holding everything else equal, the Boycott strategy is more effective than Exit in terms of requiring a lower fraction of activists to be present for technology adoption. See Internet Appendix IA.2.1 for further discussions.

4.2.3 Equilibrium under Voice

Finally, we examine Voice. Consider a scenario in which activists hold only the shares of the brown firm (and divest from investing in the green firm) at $t = 1$ and $t = 2$:

$$\begin{cases} \theta_{G1}^{\mathcal{A}} = 0 \\ \theta_{G2}^{\mathcal{A}} = 0 \end{cases}. \tag{4.6}$$

When the aggregate share of activists is more than half of the outstanding shares of the brown firm at $t = 2$, i.e., $k_2\theta_{B2}^{\mathcal{A}} > 0.5$, the activists can replace the incumbent manager of the brown firm with an activist manager who always adopts the green technology. We examine the equilibrium when activists behave in this manner as given in (4.6). As in an Exit or Boycott equilibrium, a sufficiently large activist population is necessary to make any difference in a Voice equilibrium.[24]

Proposition 4.5. *In a Voice equilibrium when $q = 0$, there are thresholds $\bar{\bar{k}}_{Voice}$ and $\hat{k}_{Voice}$ such that*

- *if $k_1 < \bar{\bar{k}}_{Voice}$, share prices will be the same as in the Benchmark equilibrium, and the green technology will not be adopted.*
- *if $\bar{\bar{k}}_{Voice} < k_1 \leq \hat{k}_{Voice}$, activists are majority shareholders of the brown firm, and the green technology will be adopted.*
- *if $k_1 > \hat{k}_{Voice}$, activists are the only shareholders of the brown firm, and the green technology will be adopted.*

We can compare the thresholds of k_1 such that the green technology is adopted under Boycott and Voice. We have already shown that for a given fraction of activists in the population, if the green technology is adopted under Exit, it will also be adopted under Boycott.

[24]Note that given the timing of proxy voting, state S_{1A} and consequently state S_{23} will not be realized.

Proposition 4.6. *With homogeneous endowments across agents, $k_1 > \frac{D_{G3}}{D_{B3}+D_{G3}}$ is necessary for the green technology to be adopted in Exit and Boycott equilibria. In a Voice equilibrium, the green technology will be adopted if $k_1 > \bar{\bar{k}}_{Voice} = \frac{1}{2} \cdot \frac{D_{B3}}{D_{G3}(1+\tau)+D_{B3}}$.*

Proposition 4.6 implies that if the brown firm is larger than the green firm, the Boycott strategy easily becomes effective by boosting the price of green consumption good. By contrast, when the green firm is larger than the brown firm, activists can easily hold the majority of the brown firm shares through the Voice strategy.

To understand the derivation of $\bar{\bar{k}}_{Voice}$, note that all agents have homogeneous endowments of shares. When the technology is not adopted, the value of the brown firm is $\frac{1}{1+\tau}D_{B3}$ since the price of brown good is $\frac{1}{1+\tau}$ and there is no time discount. $\bar{\bar{k}}_{Voice}$ is determined such that the aggregate value of share endowments of activists can buy half of the brown firm. That is, $\bar{\bar{k}}_{Voice}\left(D_{G3} + \frac{1}{1+\tau}D_{B3}\right) = \frac{1}{2}\frac{1}{1+\tau}D_{B3}$, which gives the threshold in Proposition 4.6.

When the firms are equal-sized ($D_{B3} = D_{G3}$), in an Exit equilibrium, activists have to hold 100% of the green firm's shares to have a price impact. In a Voice equilibrium, it is sufficient for activists to hold just more than 50% of the brown firm shares for the green technology to be adopted. Furthermore, note that the condition $k_1 > \frac{D_{G3}}{D_{B3}+D_{G3}}$ is *necessary* for the green technology to be adopted in an Exit and Boycott equilibria. In fact, this condition just guarantees that the Exit (4.4) or Boycott (4.5) makes the equilibrium different from the Benchmark equilibrium (Section 4.1). However, we find that k_1 needs to be sufficiently larger than $\frac{D_{G3}}{D_{B3}+D_{G3}}$ to make brown share price low enough so that the brown firm manager adopts the green technology.

So far, we have shown that with equal firm size, fewer activists are needed for the adoption of the green technology under Voice than under Exit or Boycott. However, from the perspective of activists, Voice can be costly, especially when the fraction of activists in the economy is very large. Note that when the fraction of activists is sufficiently large (e.g., $k_1 > \hat{k}$), the green technology is adopted in under Exit, Boycott as well as Voice. Under Exit or Boycott, activists do not have to divest or boycott in equilibrium. Hence, activists do not incur any cost. By contrast, under Voice, activists will boost the brown firm's share price at $t = 1$ since they will be holding 100% of the brown firm shares and therefore suffer from buying brown shares at a high price.

Example 3. In this example, we assume the same set of parameters as in the previous examples, where the two firms still have the same size. We defer the discussion of unequal firm sizes to Section 5.2.

Figure 6 plots the share prices in states S_1 and S_{21} under the Voice strategy. The activist manager for the brown firm is elected when activists hold more than 50% of the brown firm's shares, i.e., $k_1 > \bar{\bar{k}}$. As shown in Proposition 4.6, when firm sizes are equal, the Voice strategy requires the least fraction of activists to have the green technology adopted. In the example, we see that the threshold is 0.238, much lower than the one under Exit or Boycott.

More interesting is the behavior of share prices at $t = 1$ when activists own all of the brown firm's shares ($k_1 > \hat{k}$). By assumption, every agent has the same endowments of shares prior to trading at $t = 1$. As k_1 increases, the activists in aggregate will offload more endowed green shares in order to purchase the brown shares. This will create a downward pressure on the green share price while boosting the brown share price. Furthermore, to incentivize the non-activists who are indifferent toward holding brown or green shares to hold only the green shares, the return on the green shares must be high enough. The binding short-selling constraint ensures the share prices do not converge.

Figure 7 plots the utilities at $t = 1$. When $k_1 \leq \hat{k}$, agents have the same utility, although the utility is slightly higher when $k_1 \in [\bar{\bar{k}}, \hat{k}]$ due to the removal of public bad. When activists are the only shareholders of the brown firm, the utility of the (non-)activists become (higher) lower. This is because the activists have to bear the high cost of and low returns on brown shares when k_1 is large. The non-activists, on the other hand, enjoy both the benefits of high returns on green shares and no public bad, leading to higher utilities. This illustrates that even though Voice appears to be the most effective strategy, it can be very costly to activists when the technology is adopted. By contrast, when the technology is adopted under either Exit or Boycott, both types of agents have identical utilities.

4.2.4 Summary of equilibria under various strategies

From the previous analyses and numerical illustrations, we observe that the fraction of activists is crucial for determining share prices and green technology adoption (3.5) under the Exit (4.4), Boycott (4.5), and Voice (4.6) strategies. The intuition in general is that when there are too few activists in the economy, their actions will not cause any material impact on equilibrium prices. Thus, the strategy will result in the same equilibrium as the Benchmark equilibrium. When there are a sufficient number of activists and their aggregate wealth is large enough to buy out all of the green shares (Exit) or green consumption goods (Boycott), or own the majority shares of brown firm (Voice), their actions will be reflected in equilibrium prices. In the Voice strategy, this means that the activist manager will be elected and the green technology is adopted. For Exit and Boycott, however, it does not mean the green technology will be automatically adopted. The default brown firm's manager compares share prices and acts according to the technology rule (3.5). Only when the number of activists is high enough ($k_1 > \bar{\bar{k}}_{Exit}$ or $\bar{\bar{k}}_{Boycott}$), which causes a large enough equilibrium price impact, will the green technology be adopted. We summarize the threshold levels of k_1 under different strategies in Table 4.

5 Heterogeneous Endowments and Firm Sizes

5.1 Heterogeneous endowments

In this section, we assume that activists and non-activists have different endowments while setting $q = 0$. However, all agents of a given type have the same endowments. It is straightforward to see that in equilibrium all agents of same type make the same decision. Since there is no heterogeneity within each type of agents, we continue to denote the consumption and shareholdings of activists and non-activists with a superscript $\mathcal{A}$ and $\mathcal{N}$, respectively. The fraction of activists at time $t \in \{1, 2\}$ is again given by $k_t \in [0, 1]$ as before, but their endowments are no longer identical to the non-activists'. Let λ_ψ denote the ratio of the endowments of consumption goods of each activist to that of each non-activist, i.e., $\psi_t^{\mathcal{A}} = \lambda_\psi \psi_t^{\mathcal{N}}$ for $t = 0, 1,$ and 2.[25] Similarly, we let λ_θ be the ratio of the initial shares of each firm owned by each activist to that of each non-activist, i.e., $\theta_B^{\mathcal{A}} = \lambda_\theta \theta_B^{\mathcal{N}}$ and $\theta_G^{\mathcal{A}} = \lambda_\theta \theta_G^{\mathcal{N}}$. In what follows, we set $\lambda \equiv \lambda_\psi = \lambda_\theta$ for simplicity and name λ as the individual wealth ratio. The

[25] At $t = 0$, even though everyone knows about the wealth distribution, no one thinks that they will become an activist in the future. Nevertheless, we still assign labels $\mathcal{A}$ and $\mathcal{N}$ to the agents for notational convenience.

homogeneous endowment cases studied in Sections 4.2.1–4.2.3 correspond to $\lambda = 1$.

Since the intuition behind the results in the homogeneous endowment cases does not depend on homogeneity *per se*, we expect the theoretical results of the homogeneous endowment case to continue to hold for heterogeneous endowments. However, under heterogeneous endowments, as the activist population size changes, we need to adjust either the individual endowments or the aggregate endowments in a somewhat arbitrary manner, which makes the economic intuition unclear and the proofs of propositions complicated. Hence, we rely on numerical examples in this section.

We fix the total endowment of consumption goods at $t = 0, 1$, and 2 to be 1 and vary the initial wealth ratio λ. Figure 8 shows the adoption threshold $\bar{\bar{k}}$ for the Exit strategy with different levels of initial wealth. We confirm that as activists become wealthier, the fraction of activists needed to induce the manager to adopt the technology is lower, and vice versa.

Furthermore, as illustrated in Table 5, poor activists are worse off relative to the first best equilibrium than wealthy non-activists when $k_1 \in (\bar{k}, \bar{\bar{k}})$. There are two reasons for this. First, activists have to pay a higher price to purchase the green shares, which effectively lowers their consumption. Second, a poor activist only consumes a small fraction of the consumption goods, but prior to adoption of the green technology the public bad affects everyone equally. A wealthy non-activist, on the other hand, enjoys more consumption goods but is not subject to more public bad. The disutility from the public bad has a higher weight in the overall utility of a poor activist than a wealthy non-activist. Therefore, removing the public bad through the green technology will have a larger positive effect for poor activists.

5.2 Heterogeneous firm sizes

In this section, we extend the examples in Section 4.2 by allowing firms to have different sizes ($D_{B3} \neq D_{G3}$) while setting $q = 0$. Every agent still has identical endowments in shares and consumption goods. Except for the firm sizes, we use the same parameters as in Section 4.2. We fix the total output size to be the same as the previous section, i.e., $D_{B3} + D_{G3} = 1.14$, and vary the ratio of firm G's output to firm B's, denoted by ζ. Figure 9 plots the relationship between the technology adoption threshold $\bar{\bar{k}}$ and relative firm size

ζ for all three activist strategies.

There are several important observations. First, for each firm size ratio ζ, the curve for Boycott is always to the left of the curve for Exit. As a result, in regions E and F, the green technology is adopted under Boycott but not under Exit. This echos the result in Proposition 4.4 that Boycott is always more effective than Exit in terms of requiring fewer activists in the population for the green technology to be adopted. Second, for Exit and Boycott, as the green firm becomes larger, the adoption threshold also increases. This is because a larger green firm makes both Exit and Boycott more difficult to affect the equilibrium share prices as a higher aggregate wealth of activists is needed to absorb all the green shares or green consumption goods. This consequently increases the adoption threshold. For Voice, the situation is the opposite. As the brown firm becomes smaller, the brown share price also decreases, which makes it easier for activists to hold the majority of brown shares. Thus, as the green firm becomes larger (or equivalently the brown firm becomes smaller), the adoption threshold decreases. Indeed, in region B where ζ is sufficiently large, the green technology is adopted only under Voice. Finally, when the brown firm is sufficiently large, Voice becomes the least effective and has the largest $\bar{\bar{k}}$ among all three activist strategies. In region D, the green technology is adopted under both Exit and Boycott but not under Voice. These results are reflected in Proposition 4.6.

5.2.1 Numerical calibration and discussion

We calibrate the firm sizes using the proxies provided by Berk and van Binsbergen (2022), which find that the brown industry makes up 15—45% of the total US market cap, depending on how we restrict the firms to be brown. This implies that the relative size of the green firm is $\zeta = 1.2$—5.7. Using this value with other parameters as in Table 1, Exit and Boycott require $\bar{\bar{k}}$ to be 0.62–0.88.[26] Voice, on the other hand, will be the most effective strategy as it only requires $\bar{\bar{k}}$ to be 0.07—0.22.

All three strategies have limitations that we have not modeled. Exit and Voice work only for public companies, but firms may become private in response to activism. Boycott

[26] A contemporaneous work by Berk and van Binsbergen (2022) studies the effectiveness of Exit and reaches a similar conclusion.

requires that consumers are able to identify the origins of consumption goods, and substitutes for brown goods must be available. Even with limitations, activism may increase public awareness of environmental issues among both the wealthy and poor and hence increase the chance of legislative enactment of Pigouvian taxes and subsidies.[27]

6 Summary and Conclusion

We develop a model economy with a green and a brown firm, where the brown firm generates a negative externality when producing the consumption good. Agents in the economy are atomistic and cannot individually affect adoption of the green technology. However, their action in the aggregate can affect equilibrium share prices and induce the brown firm manager to adopt the green technology if and when it becomes available. Adoption depends on the number of activists in the economy, which in reality is endogenous. While we do not model this endogeneity, we characterize this dependence.

We examine Exit, Boycott, and Voice as possible strategies. We find that Exit and Boycott have much in common. A low fraction of activists has no effect on share prices, and a sufficiently high fraction of activists induces the brown firm manager to adopt the green technology. With an intermediate fraction of activists, green shares sell at a large premium relative to brown shares but the green technology is not adopted. The thresholds that the fraction of activist agents need to cross for Boycott are lower than those for Exit, suggesting that Boycott may be a more effective strategy.

Voice requires a much lower threshold than Exit and Boycott, provided that the brown firm is not too large. Nevertheless, when activists become the only shareholders of the brown firm (leading to technology adoption), they incur a significant personal cost due to initially buying brown shares at a large premium. This observation is in contrast to Exit and Boycott, under which all agents have the same utility after technology adoption. Furthermore, there are several impediments to implementing Voice in practice. For example, not all shares may have equal voting power, a few agents may control most of the votes, and it may not be

[27]In contrast to Voice, Pigouvian taxes and subsidies may lead to development of clean alternatives to brown technology. The rise of the photovoltaic solar industry is an example.

easy to get a shareholder resolution on the proxy-ballot for voting. All these will limit the effectiveness of Voice.

We find that relative firm size and the initial wealth of activists also play an important role in technology adoption. The larger the green firm is, the less effective the Exit and Boycott strategies become, since their aggregate effect on equilibrium share prices are smaller. The opposite occurs for the Voice strategy — a larger brown firm makes the strategy less effective. When activists are wealthier, their aggregate wealth and consumption will be larger and their actions have a larger impact on share prices. Therefore, all of the activist strategies become more effective.

Increasing concerns about pollution can lead to an unexpected increase in the number of activists. When the economy shifts from there being no activists to a large number of activists, green shares can trade at a huge premium to brown shares, even after accounting for emissions tax.

Reality is more complex since firms in practice may go private as a response to activist pressure. Privatization stymies both Exit and Voice strategies and would be appealing to agents who do not care about a firm's negative externality. Boycott, by contrast, does not require shares to be publicly traded. However, it assumes that agents know the source of the consumption good and the associated public bad. In practice, accurately labeling goods as green may be difficult, especially with a globally distributed supply chain.

When a sufficiently high emissions tax is imposed on brown output through legislative action, the brown firm's manager will adopt the green technology whenever it is available, leading to the socially optimal outcome. Given enough activists, all three activist strategies can also achieve the socially optimal outcome of converting the brown firm to green, but in practice, for reasons we have outlined, there may be no real alternative to legislative action. That said, legislative action requires awareness, and activism plays an important role in increasing awareness.

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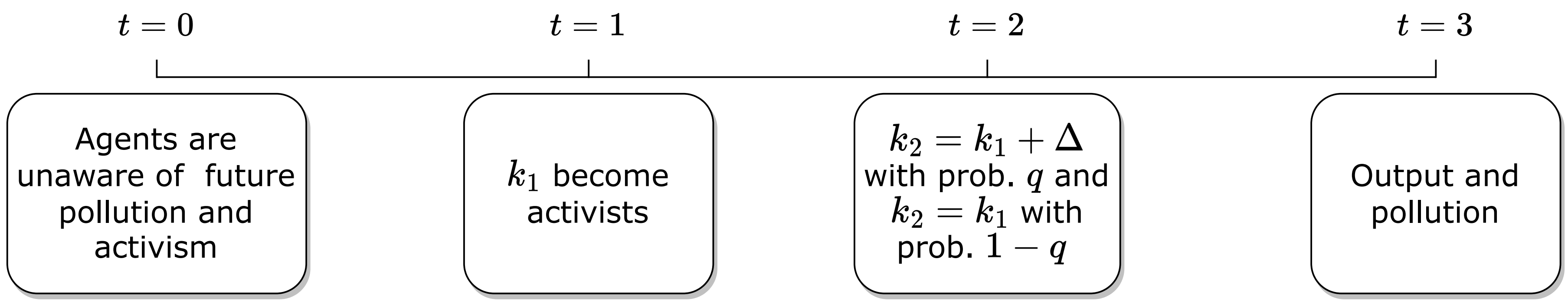


Figure 1: Timeline of the model.

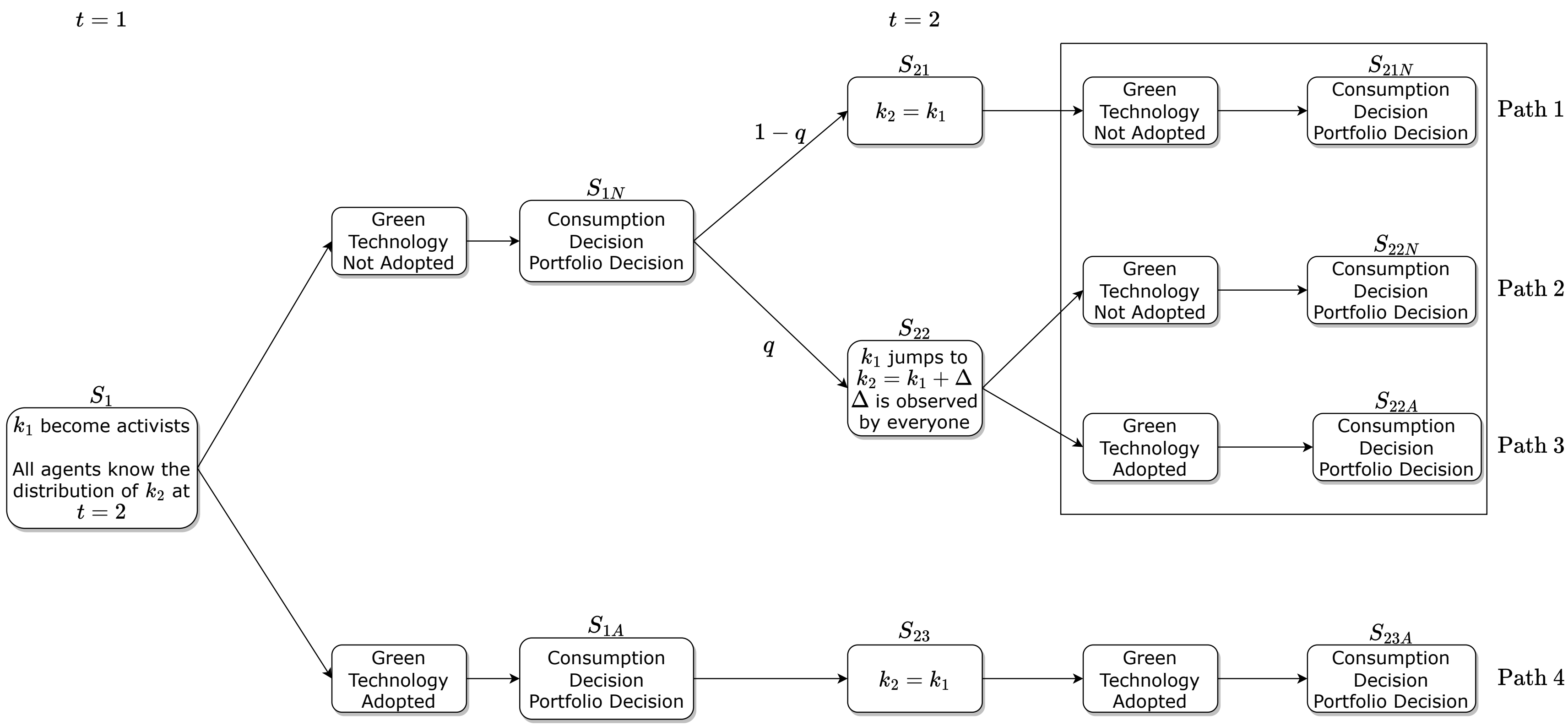


Figure 2: Events of the model at $t = 1$ and $t = 2$. The symbols S_1–S_{23A} represent the "state" of the economy. N indicates that firm B's manager does not adopt the green technology while A indicates that the manager adopts. Paths 1–4 denote possible equilibrium paths. In equilibrium, only one path will ex post be realized.

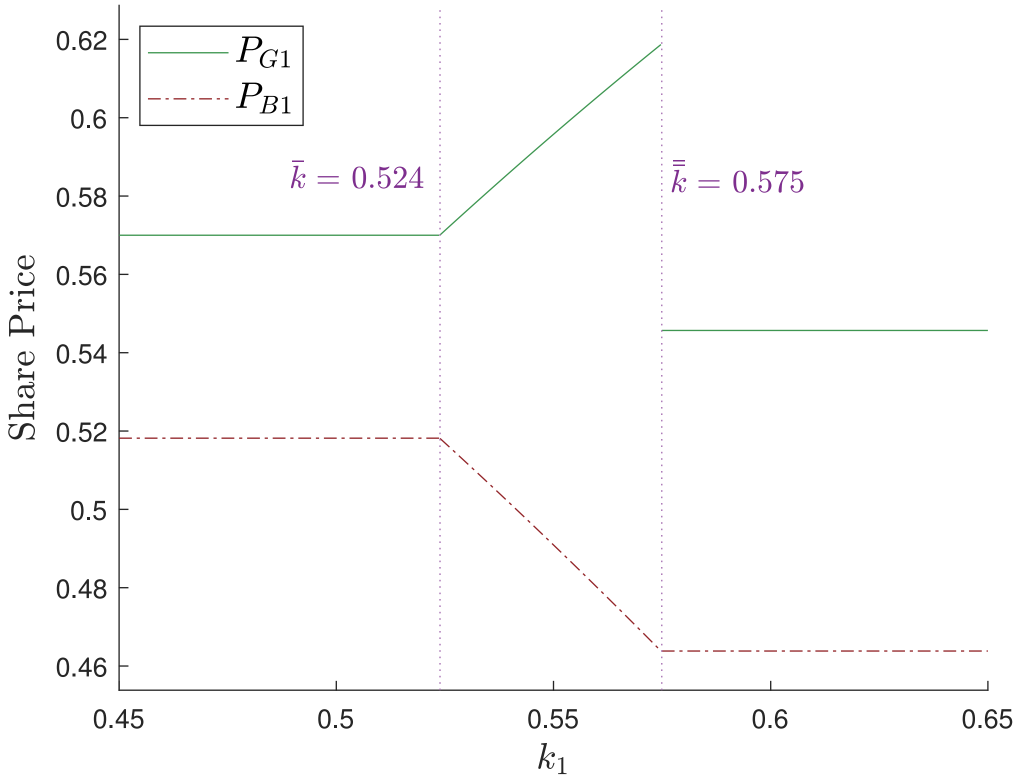
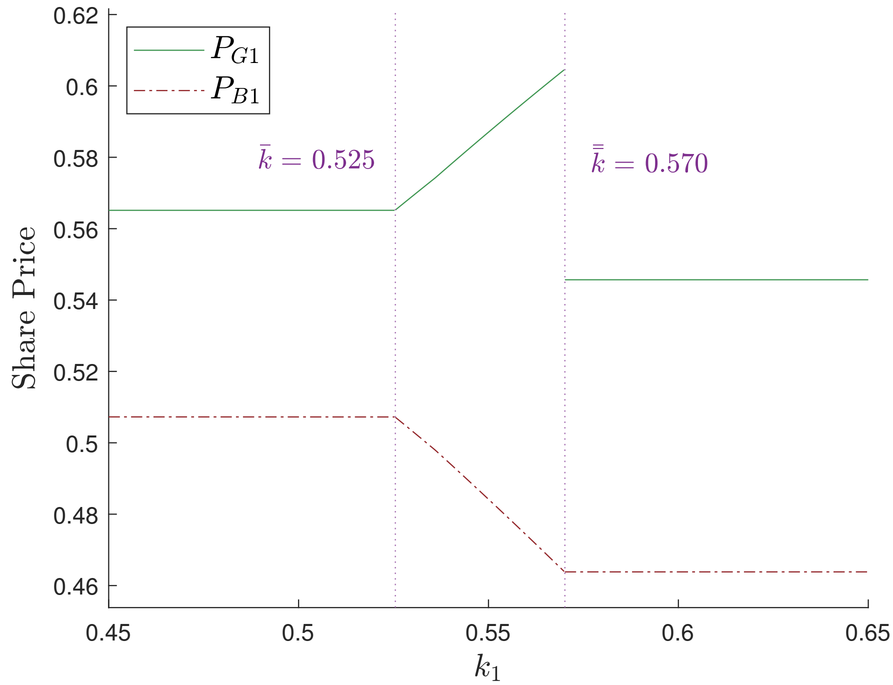
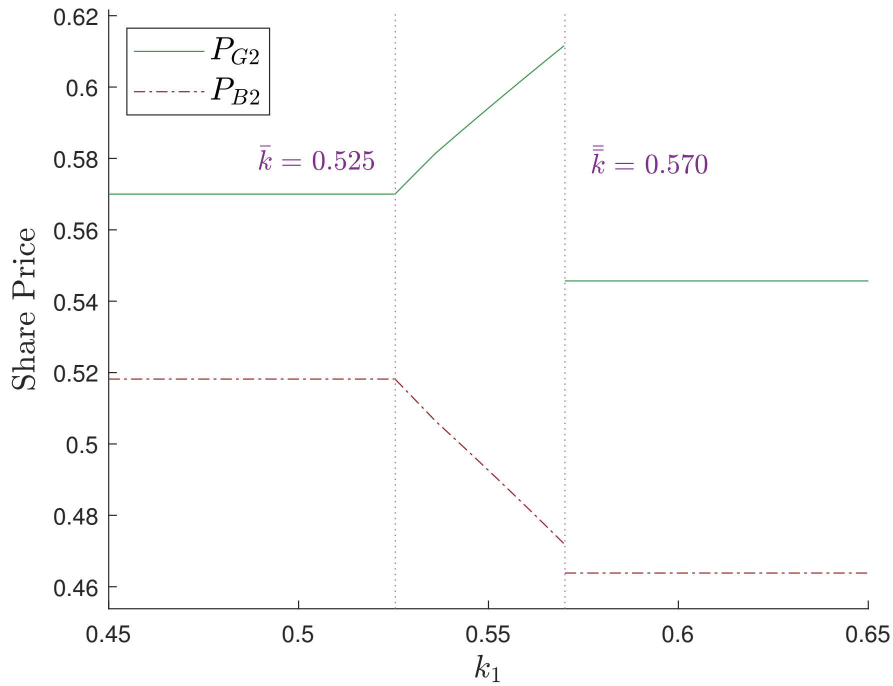


Figure 3: Exit - share prices at $t = 1$ when $q = 0$. This figure plots green and brown share prices at $t = 1$ as functions of the fraction k_1 of activists. The dotted lines denote the threshold levels of k_1, beyond which the Exit strategy either has a price impact ($\bar{k}$) or induces firm B's manager to adopt the green technology ($\bar{\bar{k}}$). Parameters are from Table 1 except that $q = 0$.



(a) Share prices at $t = 1$



(b) Share prices in states S_{21} and S_{23}

Figure 4: Exit - share prices when $q = 0.2$ and $k_2 > \bar{\bar{k}}$. This figure plots share prices at $t = 1$ and in states S_{21} and S_{23} as functions of the fraction k_1 of activists. The dotted lines denote the threshold level of k_1, beyond which the Exit strategy either has a price impact ($\bar{k}$) or induces firm B's manager to adopt the green technology ($\bar{\bar{k}}$). Parameters are from Table 1.

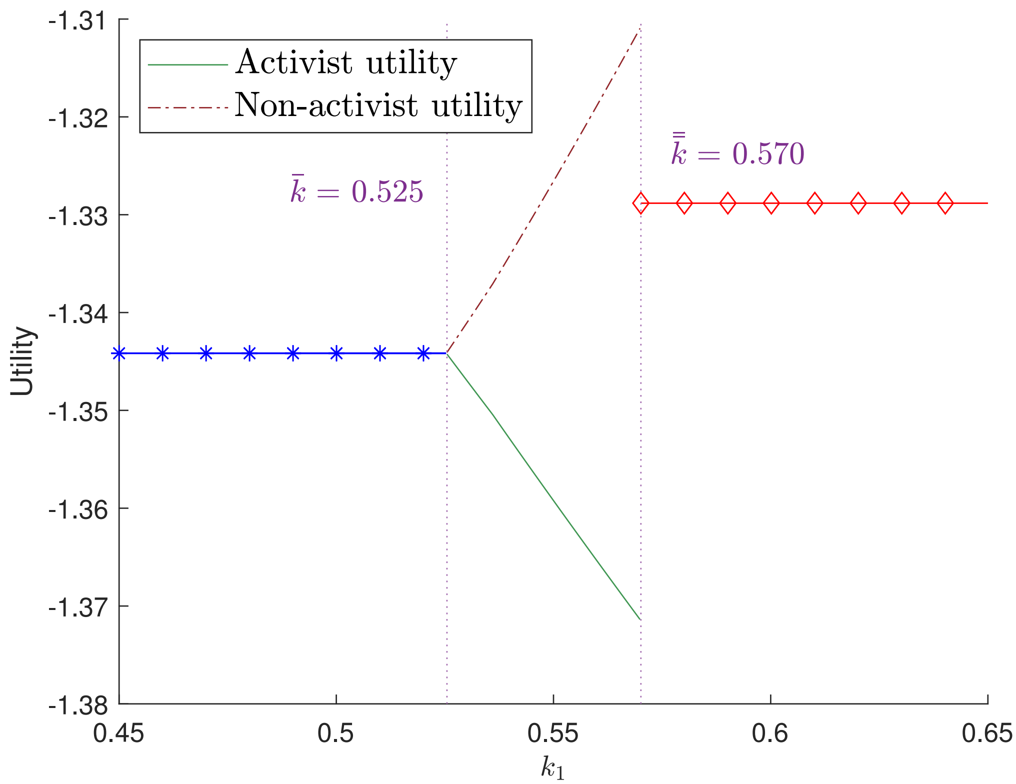
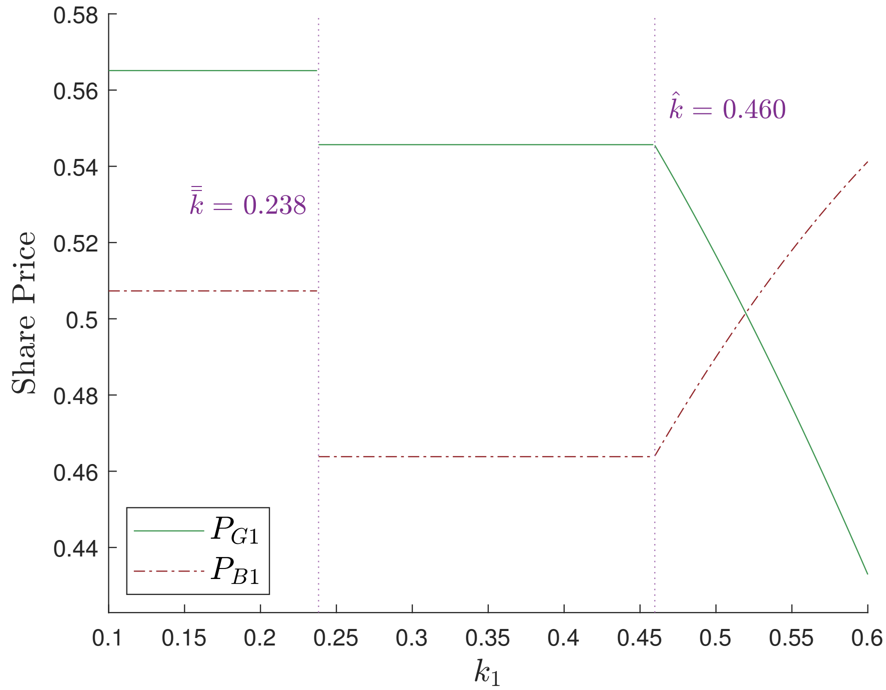
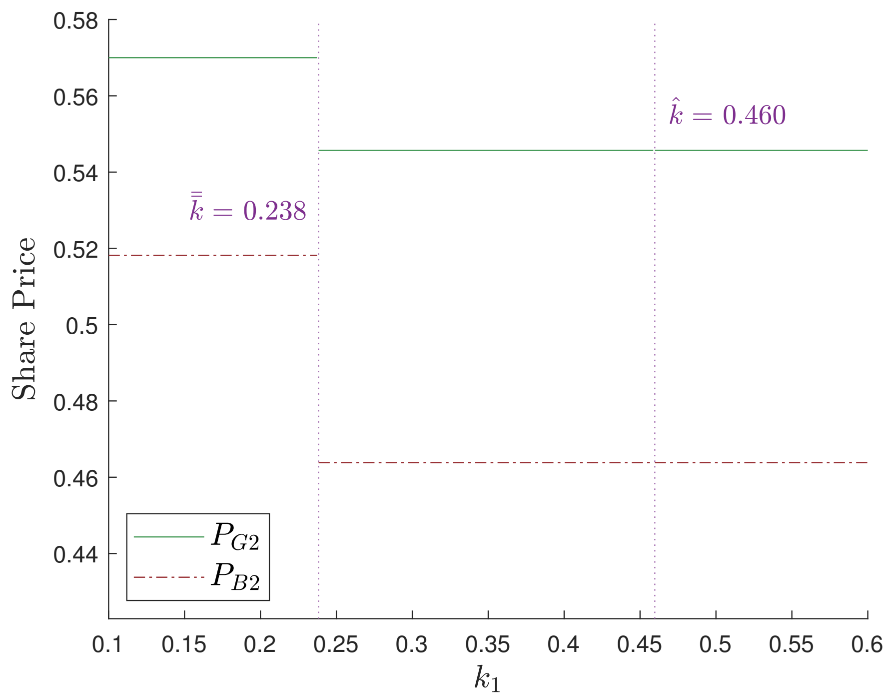


Figure 5: Exit - utility at $t = 1$ when $q = 0.2$ and $k_2 > \bar{\bar{k}}$. This figure plots utilities of individual agents at $t = 1$ as functions of the fraction k_1 of activists. The dotted lines denote the threshold levels of k_1, beyond which the Exit strategy either has a price impact ($\bar{k}$) or induces firm B's manager to adopt the green technology ($\bar{\bar{k}}$). The line marked by stars is the utility level for both types of agents when $k_1 \leq \bar{k}$. The solid and dash-dotted lines indicate the utility levels for activists and non-activists, respectively, when $k_1 \in [\bar{k}, \bar{\bar{k}}]$. The line marked by diamonds is the utility level for both types of agents when $k_1 > \bar{\bar{k}}$. Parameters are from Table 1.



(a) Share prices at $t = 1$



(b) Share prices in state S_{21}

Figure 6: Voice - share prices when $q = 0.2$ and $k_2 > \bar{\bar{k}}$. This figure plots share prices at $t = 1$ and in state S_{21} as functions of the fraction k_1 of activists. The dotted lines denote the threshold level of k_1, beyond which the activists as a whole either hold the majority but not all of firm B's shares ($\bar{\bar{k}}$) or all of firm B's shares ($\hat{k}$). Parameters are from Table 1.

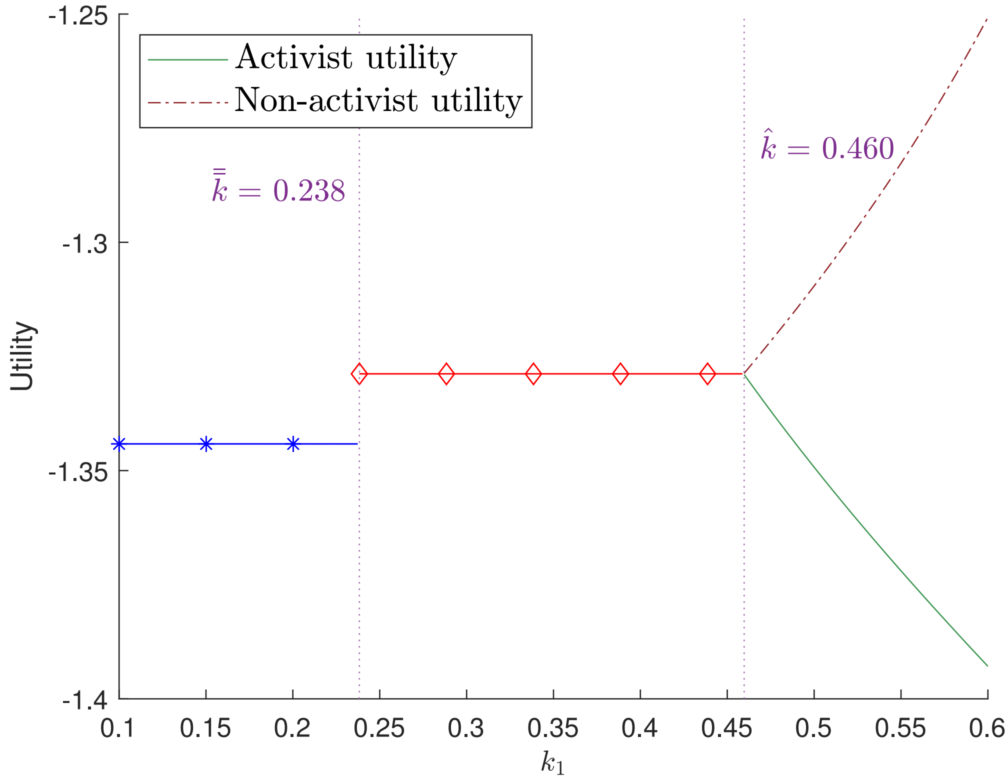


Figure 7: Voice - utility at $t = 1$ when $q = 0.2$ and $k_2 > \bar{\bar{k}}$. This figure plots utilities of individual agents at $t = 1$ as functions of the fraction k_1 of activists. The dotted lines denote the threshold level of k_1, beyond which activists as a whole either hold the majority but not all of firm B's shares ($\bar{\bar{k}}$) or all of firm B's shares ($\hat{k}$). The line marked by stars is the utility level for both types of agents when $k_1 \leq \bar{\bar{k}}$. The line marked by diamonds is the utility level for both types of agents when $k_1 \in [\bar{\bar{k}}, \hat{k}]$. The solid and dash-dotted lines indicate the utility levels for activists and non-activists, respectively, when $k > \hat{k}$. Parameters are from Table 1.

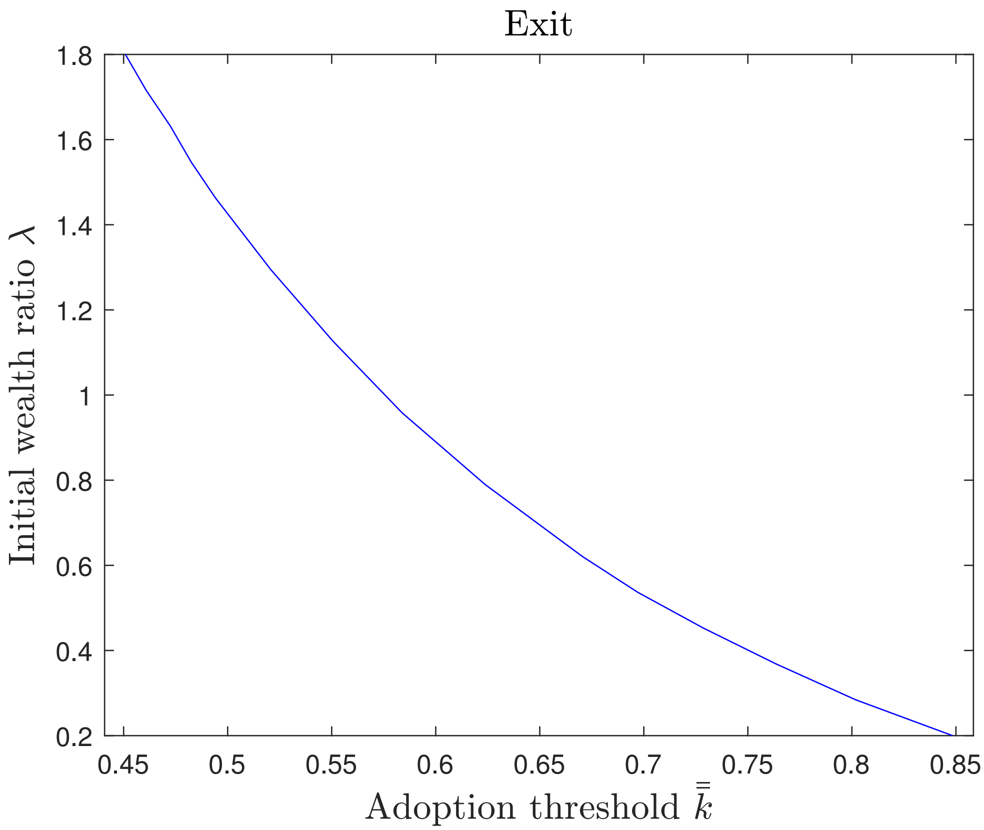


Figure 8: Exit - adoption threshold with heterogeneous wealth when $q = 0$. This figure plots the adoption threshold $\bar{\bar{k}}$ for the Exit strategy as λ varies, where λ is defined to be the ratio of an activist's initial wealth to a non-activist's. Parameters are from Table 1 except that individual wealth ratio varies and $q = 0$.

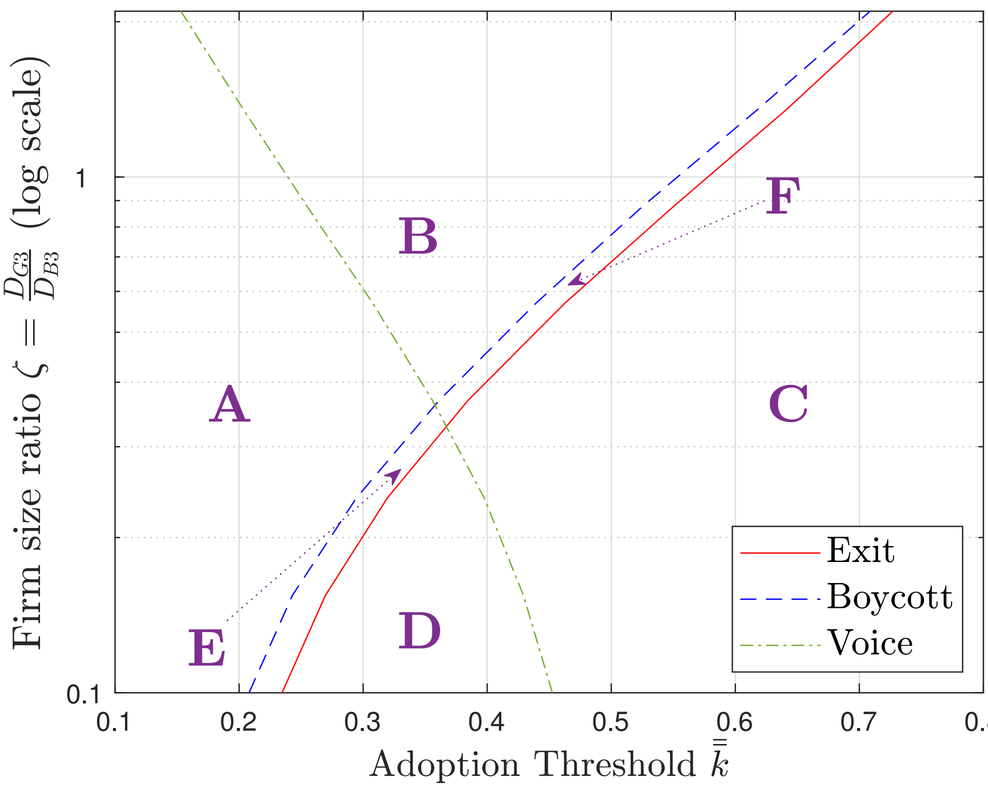


Figure 9: Comparison of activist strategies for different firm sizes when $q = 0$. This figure plots the adoption threshold $\bar{\bar{k}}$ for each of the three activist strategies as the firm size ratio $\zeta = \frac{D_{G3}}{D_{B3}}$ varies. The vertical axis is on a log scale. For Exit, the green technology is adopted in regions C and D. For Boycott, the technology is adopted in regions C, D, E, and F. For Voice, the technology is adopted in regions B, C, and F. In region A, none of the activist strategies adopts the technology. Parameters are from Table 1 except that individual firm size varies and $q = 0$.

Table 1: Base-case parameters for examples. This table reports the baseline parameters used in the numerical examples. Endowments in the table refer to individual endowments of a type-i agent, where $i \in \{\mathcal{A}, \mathcal{N}\}$. All agents of same type receive identical endowments.

Description	Parameter	Value
Individual endowment of consumption good, $t = 0, 1, 2$	$\psi_{i0}, \psi_{i1}, \psi_{i2}, i \in [0, 1]$	1
Individual endowment of brown and green shares, $t = 0$	$\theta_{iB}, \theta_{iG}, i \in [0, 1]$	1
Aggregate endowment of consumption good, $t = 0, 1, 2$	ψ_0, ψ_1, ψ_2	1
Aggregate endowment of brown and green shares, $t = 0$	θ_B, θ_G	1
CARA coefficient	A	0.8
Emissions tax	τ	0.1
Probability of an increase in activist population	q	0.2
Output reduction from adopting green technology	η	0.15
Scale factor for public bad	δ	0.281
Dividends from brown and green firms, $t = 3$	D_{B3}, D_{G3}	0.57

Table 2: Exit - share prices when $q = 0.2$ and $k_2 > \bar{\bar{k}}$ for $k_1 \leq \bar{k}$ and $k_1 \geq \bar{\bar{k}}$. This table reports share prices (p^s_{Gt} and p^s_{Bt}) for small and large k_1. Parameters are from Table 1.

	$k_1 \leq \bar{k}$		$k_1 \geq \bar{\bar{k}}$	
State	p^s_{Gt}	p^s_{Bt}	p^s_{Gt}	p^s_{Bt}
S_0	0.510	0.510	0.510	0.510
S_1	0.565	0.507	0.546	0.464
S_{21}	0.570	0.518	0.546	0.464
S_{22}	0.546	0.464	0.546	0.464
S_{31N}	0.570	0.518	-	-
S_{31A}	-	-	0.570	0.485
S_{32A}	0.570	0.485	0.570	0.485

Table 3: : Exit - time series of equilibrium share prices and returns when $q = 0.2$ and $k_2 > \bar{\bar{k}}$. This table reports share prices (p^s_{Gt} and p^s_{Bt}) and one-period realized return (r^s_{Gt} and r^s_{Bt}) for two values of k_1. The returns are reported in percentage. Paths 1 and 3 refer to the equilibrium paths in Figure 2. Parameters are from Table 1.

Panel A: $\bar{k} < k_1 = 0.53 \ll \bar{\bar{k}}$

	Path 1: $k_2 = k_1$				Path 3: $k_2 = k_1 + \Delta$			
State	p^s_{Gt}	p^s_{Bt}	$r^s_{Gt}\%$	$r^s_{Bt}\%$	p^s_{Gt}	p^s_{Bt}	$r^s_{Gt}\%$	$r^s_{Bt}\%$
S_0	0.510	0.510	-	-	0.510	0.510	-	-
S_1	0.569	0.503	11.680	-1.253	0.569	0.503	11.680	-1.253
S_{21}	0.575	0.513	1.029	1.966	-	-	-	-
S_{22}	-	-	-	-	0.546	0.464	-4.119	-7.828
S_{31N}	0.570	0.518	-0.867	0.988	-	-	-	-
S_{32A}	-	-	-	-	0.570	0.485	4.456	4.456

Panel B: $\bar{k} \ll k_1 = 0.57 < \bar{\bar{k}}$

	Path 1: $k_2 = k_1$				Path 3: $k_2 = k_1 + \Delta$			
State	p^s_{Gt}	p^s_{Bt}	$r^s_{Gt}\%$	$r^s_{Bt}\%$	p^s_{Gt}	p^s_{Bt}	$r^s_{Gt}\%$	$r^s_{Bt}\%$
S_0	0.510	0.510	-	-	0.510	0.510	-	-
S_1	0.605	0.463	18.635	-8.975	0.605	0.463	18.635	-8.975
S_{21}	0.612	0.472	1.172	1.735	-	-	-	-
S_{22}	-	-	-	-	0.546	0.464	-9.741	-0.008
S_{31N}	0.570	0.518	-6.811	9.803	-	-	-	-
S_{32A}	-	-	-	-	0.570	0.485	4.456	4.456

Table 4: Thresholds comparison when $q = 0.2$ and $k_2 > \bar{\bar{k}}$. This table reports the thresholds $\bar{k}, \bar{\bar{k}}$ and $\hat{k}$ for different activist strategies. Parameters are from Table 1.

	$\bar{k}$	$\bar{\bar{k}}$	$\hat{k}$
Exit	0.525	0.570	-
Boycott	0.500	0.554	-
Voice	-	0.238	0.461

Table 5: Utility comparison when $q = 0$. This table reports utility differences from the first best equilibrium for individual agents when k_1 is just below $\bar{\bar{k}}$. Parameters are from Table 1 except that individual wealth ratio is $\lambda = 0.75$ and $q = 0$.

	Activist	Non-activist
$\lambda = 0.75$	-0.046	0.014

A Proofs

For the proofs in this section as well as in the Internet Appendix IA.1, we refer to a more detailed timeline in Figure 10.

We first prove Proposition 4.1. For that purpose, we develop several lemmas. Lemma A.1 identifies the prices of brown consumption goods at $t = 3$ for different states. The share prices of the brown firm in states $\{S_{21A}, S_{22A}, S_{23A}\}$ and $\{S_{21N}, S_{22N}\}$ are examined in Lemmas A.2 and A.3, respectively. Then, we prove Proposition 4.1 using Lemmas A.1–A.3. Furthermore, Lemma A.8 characterizes the Benchmark equilibrium, the properties of which will be used for proving subsequent propositions.

We introduce some notations. The discount factor between $t = v$ and $t = 3$ is

$$M_{\mathtt{AD}}\left(c_v, c_3\right) = e^{Ac_v - Ac_3} \tag{A.1}$$

$$M_{\mathtt{NAD}}\left(c_v, c_3\right) = e^{Ac_v - A\left(c_3 - \delta D_{B3}\mathbf{1}_{v\neq 0}\right)}, \tag{A.2}$$

where c_v and c_3 are consumption at $t = v$ and 3, $M_{\mathtt{AD}}\left(c_v, c_3\right)$ represents the marginal rate of substitution between states S_v and $S_3 \in \{S_{31A}, S_{32A}, S_{33A}\}$, when the green technology is adopted, and $M_{\mathtt{NAD}}\left(c_v, c_3\right)$ represents the discount factor between states S_v and $S_3 \in \{S_{31N}, S_{32N}\}$, when the green technology is not adopted. Also, we define the aggregate endowment as ψ :

$$\psi = \int_i \psi_i di. \tag{A.3}$$

Recall that there are three types of agents $i \in \{\mathcal{A}, \mathcal{N}, \mathcal{R}\}$. $\mathcal{A}$ represents activist at time 1. $\mathcal{N}$ represents non-activist at time 2. $\mathcal{R}$ represents reformer who was non-activist at time 1 but becomes activist at time 2 with a probability q.

Lemma A.1. *In the Benchmark equilibrium, Exit equilibrium, and Voice equilibrium, it holds that $p^c_{B3} = \frac{1}{1+\tau}$ in states $S_3 \in \{S_{31N}, S_{32N}\}$*
In the Benchmark equilibrium, Exit equilibrium, Voice equilibrium and Boycott equilibrium, it holds that $p^c_{B3} = 1$ in state $S_3 \in \{S_{31A}, S_{32A}, S_{33A}\}$.

Proof of Lemma A.1 We verify the first statement. When the green technology is not adopted or $S_3 \in \{S_{31N}, S_{32N}\}$, the consumption tax on the brown goods is active and there is no restriction on buying consumption goods from the brown firm in the Benchmark equilibrium, Exit equilibrium, or Voice equilibrium. Hence, a agent i maximizes $c_{iG3} + c_{iB3}$ in (3.4). If $p^c_{B3} > \frac{1}{1+\tau}$, agent i will not consume any brown consumption goods. If $p^c_{B3} < \frac{1}{1+\tau}$, agent i will not consume any green consumption goods. Hence, the market clearing conditions for consumption goods cannot be satisfied if $p^c_{B3} \neq \frac{1}{1+\tau}$. Thus, the first statement is true.

We move to the second statement. When the green technology is adopted or $S_3 \in \{S_{31A}, S_{32A}, S_{33A}\}$, there is no restriction on buying consumption goods from firm B and there is no consumption tax. Hence, if $p^c_{B3} \neq 1$, the market clearing condition for consumption goods cannot hold. Then, the second statement is true. This completes the proof of the Lemma. $\square$

Lemma A.2. *Consider the state $S_2 \in \{S_{21A}, S_{22A}, S_{23A}\}$. In the Benchmark equilibrium, Exit equilibrium, Boycott equilibrium, and Voice equilibrium, the equilibrium price p^s_{B2} is given by*

$$p^s_{B2} = M_{\mathtt{AD}}\left(\psi, D_{B3}\left(1-\eta\right) + D_{G3}\right) D_{B3},$$

where the discount factor $M_{\mathtt{AD}}$ is defined by (A.1). Also, it holds that

$$\frac{dU_{i2}\left(\theta_{iG1}, \theta_{iB1}\right)}{d\theta_{iG1}} = Ae^{-Ac_{i2}} p^s_{B2},$$

where $U_{i2}\left(\theta_{iG1}, \theta_{iB1}\right)$ is the maximal utility and c_{i2} is the equilibrium consumption.

Proof of Lemma A.2 Note that when $S_2 \in \{S_{21A}, S_{22A}, S_{23A}\}$, it holds that $S_3 \in \{S_{31A}, S_{32A}, S_{33A}\}$. Hence, from Lemma A.1, $p^c_{B3} = 1$. Then, U_{i3} in (3.4) simplifies to

$$U_{i3}\left(\theta_{iG2}, \theta_{iB2}\right) = -e^{-A\left(\theta_{iG2} D_{G3} + \theta_{iB2} D_{B3}(1-\eta)\right)},$$

which in conjunction with U_{i2} in (3.3) implies that

$$U_{i2}\left(\theta_{iG1}, \theta_{iB1}\right) = \max_{\theta_{iG2}, \theta_{iB2}, c_{i2}} \left\{-e^{-Ac_{i2}} - e^{-A\left(\theta_{iG2} D_{G3} + \theta_{iB2} D_{B3}(1-\eta)\right)}\right\},$$

subject to the budget constraint:

$$c_{i2} = \theta_{iG1} p^s_{G2} + \theta_{iB1} p^s_{B2} + \psi_{i2} - \theta_{iG2} p^s_{G2} - \theta_{iB2} p^s_{B2}.$$

Then, the FOC with respect to θ_{iB2} yields

$$-e^{-Ac_{i2}} p^s_{B2} + (1-\eta) D_{B3} e^{-A\left(\theta_{iG2} D_{G3} + \theta_{iB2} D_{B3}(1-\eta)\right)} = 0,$$

which gives

$$p^s_{B2} = e^{Ac_{i2} - A\left(\theta_{iG2} D_{G3} + \theta_{iB2} D_{B3}(1-\eta)\right)} D_{B3}\left(1-\eta\right).$$

Hence,

$$\log\left(\frac{p^s_{B2}}{D_{B3}\left(1-\eta\right)}\right) = Ac_{i2} - A\left(\theta_{iG2} D_{G3} + \theta_{iB2} D_{B3}\left(1-\eta\right)\right),$$

which, integrated over i, gives

$$\begin{aligned}\log\left(\frac{p^s_{B2}}{D_{B3}\left(1-\eta\right)}\right) &= \int_{i\in[0,1]} Ac_{i2} - A\left(\theta_{iG2} D_{G3} + \theta_{iB2} D_{B3}\left(1-\eta\right)\right) di \\ &= \psi - A\left(D_{B3}\left(1-\eta\right) + D_{G3}\right),\end{aligned}$$

where the last equality is from the market clearing condition. Hence, with (A.1), the first claim holds. The second claim directly follows from the Envelope theorem. This completes the proof of the lemma. ☐

Lemma A.3. *Consider state $S_2 \in \{S_{21N}, S_{22N}\}$. In the Benchmark equilibrium and Voice* *equilibrium, the equilibrium price* p^s_{B2} *is given by*

$$p^s_{B2} = M_{\texttt{NAD}}\left(\psi, D_{B3} + D_{G3}\right) D_{B3} \frac{1}{1+\tau},$$

where the discount factor $M_{\texttt{NAD}}$ *is defined by (A.2). Also, it holds that*

$$\frac{dU_{i2}\left(\theta_{iG1}, \theta_{iB1}\right)}{d\theta_{iG1}} = Ae^{-Ac_{i2}} p^s_{B2},$$

where $U_{i2}\left(\theta_{iG1}, \theta_{iB1}\right)$ *is the maximal utility and* c_{i2} *is the equilibrium consumption.*

Proof of Lemma A.3 The logic is similar to the proof of Lemma A.2. Note that when $S_2 \in \{S_{21N}, S_{22N}\}$, it holds that $S_3 \in \{S_{31N}, S_{32N}\}$. Hence, from Lemma A.1, $p^c_{B3} = \frac{1}{1+\tau}$. Hence, U_{i3} in (3.4) simplifies to

$$U_{i3}\left(\theta_{iG2}, \theta_{iB2}\right) = -e^{-A\left(\theta_{iG2} D_{G3} + \theta_{iB2} \frac{1}{1+\tau} D_{B3} + \frac{\tau}{1+\tau} D_{B3} - \delta D_{B3}\right)},$$

which in conjunction with U_{i2} in (3.3) that

$$U_{i2}\left(\theta_{iG1}, \theta_{iB1}\right) = \max_{\theta_{iG2}, \theta_{iB2}, c_{i2}} \left\{-e^{-Ac_{i2}} - e^{-A\left(c_{i3} - \delta D_{B3}\right)}\right\},$$

subject to the budget constraints:

$$\begin{aligned}
c_{i2} &= \theta_{iG1} p^s_{G2} + \theta_{iB1} p^s_{B2} + \psi_{i2} - \theta_{iG2} p^s_{G2} - \theta_{iB2} p^s_{B2} \\
c_{i3} &= \theta_{iG2} D_{G3} + \theta_{iB2} \frac{D_{B3}}{1+\tau} + \frac{\tau}{1+\tau} D_{B3}.
\end{aligned}$$

Then, the FOC with respect to θ_{iB2} yields

$$-e^{-Ac_{i2}} p^s_{B2} + \frac{D_{B3}}{1+\tau} e^{-A\left(c_{i3} - \delta D_{B3}\right)} = 0,$$

which gives

$$p^s_{B2} = e^{Ac_{i2} - A\left(\theta_{iG2} D_{G3} + \theta_{iB2} \frac{D_{B3}}{1+\tau} + \frac{\tau}{1+\tau} D_{B3} - \delta D_{B3}\right)} \frac{D_{B3}}{1+\tau}.$$

Integrating the above over i gives

$$\begin{aligned}
\log\left(\frac{p^s_{B2}\left(1+\tau\right)}{D_{B3}}\right) &= \int_{i\in[0,1]} Ac_{i2} - A\left(\theta_{iG2} D_{G3} + \theta_{iB2} \frac{D_{B3}}{1+\tau} + \frac{\tau}{1+\tau} D_{B3} - \delta D_{B3}\right) di \\
&= A\psi - A\left(D_{B3}\left(1-\delta\right) + D_{G3}\right),
\end{aligned}$$

where the last equality is from the market clearing conditions. Hence, with (A.2), the first claim holds. The second claim directly follows from the Envelope theorem. This completes the proof of the lemma. □

Lemma A.4. *In the Benchmark equilibrium, consider a state at* $t = 2$ *where the green* *technology is not adopted at $t = 1$. Then, the green technology is not adopted at $t = 2$ by the value maximizing manager if $\tau < \overline{\tau}$, where $\overline{\tau}$ is given by (4.1).*

Proof of Lemma A.4 Let $p^s_{B2}(S_2)$ denote the share prices of the brown firm in states $S_2 \in \{S_{21A}, S_{21N}, S_{22A}, S_{22N}\}$. Lemmas A.2 and A.3 imply that

$$\frac{p^s_{B2}(S_{21N})}{p^s_{B2}(S_{21A})} = \frac{p^s_{B2}(S_{22N})}{p^s_{B2}(S_{22A})} = \frac{M_{\mathtt{NAD}}(\psi, D_{B3} + D_{G3}) \frac{D_{B3}}{1+\tau}}{M_{\mathtt{AD}}(\psi, D_{B3}(1-\eta) + D_{G3}) D_{B3}(1-\eta)}$$
$$= \frac{e^{A(\delta-\eta)D_{B3}}}{(1-\eta)(1+\tau)} > 1,$$

where the second equality is from (A.1) and (A.2), and the inequality is from $\tau < \overline{\tau}$ and (4.1). According to the technology adoption rule (3.5), the non-activist manager does not adopt the green technology at $t = 2$. This completes the proof of the lemma. □

Lemma A.5. *In the Benchmark equilibrium, the equilibrium price p^s_{B1} in state $S_1 = S_{1A}$ is given by*

$$p^s_{B1} = M_{\mathtt{AD}}(\psi, D_{B3}(1-\eta) + D_{G3}) D_{B3},$$

where the discount factor $M_{\mathtt{AD}}$ is defined by (A.1).

Proof of Lemma A.5 Note that when $S_1 = S_{1A}$, it holds that $S_2 = S_{23A}$. Then, from (3.2), we have that

$$U_{i1}(\theta_{iG0}, \theta_{iB0}) = \max_{\theta_{iG1}, \theta_{iB1}, c_{i1}} \left\{ -e^{-Ac_{i1}} + U_{i2}(\theta_{iG1}, \theta_{iB1}) \right\},$$

subject to the budget constraint:

$$c_{i1} = \theta_{iG0} p^s_{G1} + \theta_{iB0} p^s_{B1} + \psi_{i1} - \theta_{iG1} p^s_{G1} - \theta_{iB1} p^s_{B1}.$$

Then, the FOC with respect to θ_{iB1} yields

$$-e^{-Ac_{i1}} A p^s_{B1} + \frac{dU_{i2}(\theta_{iG1}, \theta_{iB1})}{d\theta_{iG1}} = 0,$$

which, in conjunction with $\frac{dU_{i2}(\theta_{iG1}, \theta_{iB1})}{d\theta_{iG1}}$ in Lemma A.2, gives

$$-Ae^{-Ac_{i1}} p^s_{B1} + Ae^{-Ac_{i2}} p^s_{B2} = 0.$$

Hence,

$$\log \frac{p^s_{B1}}{p^s_{B2}} = -A(c_{i2} - c_{i1}),$$

which, integrated over i, gives

$$\log \frac{p^s_{B1}}{p^s_{B2}} = -A(\psi - \psi) = 0,$$

which along with p^s_{B2} from Lemma A.2, yields

$$p^s_{B1} = p^s_{B2} = M_{\texttt{AD}}\left(\psi, D_{B3}\left(1-\eta\right) + D_{G3}\right) D_{B3}.$$

This completes the proof of the lemma. $\square$

Lemma A.6. *In the Benchmark equilibrium, the equilibrium price p^s_{B1} in state $S_1 = S_{1N}$ is given by*

$$p^s_{B1} = M_{\texttt{NAD}}\left(\psi, D_{B3} + D_{G3}\right) D_{B3} \frac{1}{1+\tau},$$

where the discount factor $M_{\texttt{NAD}}$ is defined by (A.2).

Proof of Lemma A.6 Note that when $S_1 = S_{1N}$, it holds that $S_2 \in \{S_{21N}, S_{21A}, S_{22N}, S_{22A}\}$. Furthermore, from Lemma A.4, we can restrict $S_2 \in \{S_{21N}, S_{22N}\}$. Also, note that there is no difference across $S_2 \in \{S_{21N}, S_{22N}\}$ in the benchmark equilibrium. Then, from (3.2), we have that

$$U_{i1}\left(\theta_{iG0}, \theta_{iB0}\right) = \max_{\theta_{iG1}, \theta_{iB1}, c_{i1}} \left\{-e^{-Ac_{i1}} + U_{i2}\left(\theta_{iG1}, \theta_{iB1}\right)\right\},$$

subject to the budget constraint:

$$c_{i1} = \theta_{iG0} p^s_{G1} + \theta_{iB0} p^s_{B1} + \psi_{i1} - \theta_{iG1} p^s_{G1} - \theta_{iB1} p^s_{B1}.$$

Then, the FOC with respect to θ_{iB1} yields

$$-e^{-Ac_{i1}} A p^s_{B1} + \frac{dU_{i2}\left(\theta_{iG1}, \theta_{iB1}\right)}{d\theta_{iG1}} = 0,$$

which, in conjunction with $\frac{dU_{i2}(\theta_{iG1}, \theta_{iB1})}{d\theta_{iG1}}$ in Lemma A.3, gives

$$-Ae^{-Ac_{i1}} p^s_{B1} + Ae^{-Ac_{i2}} p^s_{B2} = 0.$$

Hence,

$$\log \frac{p^s_{B1}}{p^s_{B2}} = -A\left(c_{i2} - c_{i1}\right),$$

which, integrated over i, gives

$$\log \frac{p^s_{B1}}{p^s_{B2}} = -A\left(\psi_2 - \psi_1\right) = 0,$$

which, along with Lemma A.3, yields

$$p^s_{B1} = p^s_{B2} = M_{\texttt{NAD}}\left(\psi_1, D_{B3} + D_{G3}\right) D_{B3} \frac{1}{1+\tau}.$$

This completes the proof of the lemma. $\square$

Lemma A.7. *In the Benchmark equilibrium, the green technology is not adopted at $t = 1$ by the value maximizing manager if $\tau < \overline{\tau}$, where $\overline{\tau}$ is given by* (4.1)

Proof of Lemma A.7 Let $p^s_{B1}(S_1)$ denote the share prices of the brown firm in states $S_1 \in \{S_{1N}, S_{1A}\}$. Lemmas A.5 and A.6 imply that

$$\begin{aligned}\frac{p^s_{B1}(S_{1N})}{p^s_{B1}(S_{1A})} &= \frac{M_{\texttt{NAD}}(\psi, D_{B3}+D_{G3})\frac{D_{B3}}{1+\tau}}{M_{\texttt{AD}}(\psi, D_{B3}(1-\eta)+D_{G3})\, D_{B3}(1-\eta)} \\ &= \frac{e^{A(\delta-\eta)D_{B3}}}{(1-\eta)(1+\tau)} > 1,\end{aligned}$$

where the second equality is from (A.1) and (A.2), and the inequality is from $\tau < \overline{\tau}$ and (4.1). According to the technology adoption rule (3.5), the non-activist manager does not adopt the green technology at $t = 1$.

Proof of Proposition 4.1 The proposition directly follows from Lemmas A.4 and A.7. □

Now, we are ready to characterize the Benchmark equilibrium. Let $\left(\theta^{\mathcal{A}}_{G0}, \theta^{\mathcal{A}}_{B0}, \psi^{\mathcal{A}}_1, \psi^{\mathcal{A}}_2\right)$ and $\left(\theta^{\mathcal{N}}_{G0}, \theta^{\mathcal{N}}_{B0}, \psi^{\mathcal{N}}_1, \psi^{\mathcal{N}}_2\right)$ denote the share and consumption endowments of a representative activist and non-activist agent, respectively. Note that $\psi^{\mathcal{A}} = \psi^{\mathcal{A}}_1 = \psi^{\mathcal{A}}_2$ and $\psi^{\mathcal{N}} = \psi^{\mathcal{N}}_1 = \psi^{\mathcal{N}}_2$ and that we use superscript of $\mathcal{A}$ and $\mathcal{N}$ even in the Benchmark equilibrium for the ease of comparison to the equilibrium where activist agents actively participates in the market .

Lemma A.8. *Fix the portfolio holdings of activists at $t = 0$, $\left(\theta^{\mathcal{A}}_{G0}, \theta^{\mathcal{A}}_{B0}\right)$. Then, we have the following results for the Benchmark equilibrium.*
(i) The equilibrium consumption vector of activist agents $\left(c^{\mathcal{A}}_1, c^{\mathcal{A}}_2, c^{\mathcal{A}}_{G3} + c^{\mathcal{A}}_{B3}\right)$ satisfies

$$\left(c^{\mathcal{A}}_1, c^{\mathcal{A}}_2, c^{\mathcal{A}}_{G3} + c^{\mathcal{A}}_{B3}\right) = (\psi, \psi, D_{G3} + D_{B3}) + \gamma^{\mathcal{A}} \cdot (1, 1, 1), \tag{A.4}$$

for some constant $\gamma^{\mathcal{A}}$.
(ii) The equilibrium share prices of $(p^s_{Gt}, p^s_{Bt})$are given by

$$\begin{aligned} p^s_{Gt} &= M_{\texttt{NAD}}(\psi, D_{G3} + D_{B3})\, D_{G3}, \ \ t = 1, 2, \\ p^s_{Bt} &= p^s_{Gt}\frac{D_{B3}}{D_{G3}(1+\tau)}, \ \ t = 1, 2, \end{aligned} \tag{A.5}$$

where $M_{\texttt{NAD}}$ is given by (A.2).
(iii) $\gamma^{\mathcal{A}}$ in (A.4) satisfy

$$\gamma^{\mathcal{A}} = \frac{\left(\theta^{\mathcal{A}}_{G0} - 1\right)p^s_{G1} + \left(\theta^{\mathcal{A}}_{B0} - 1\right)p^s_{B1} + \left(\psi^{\mathcal{A}} - \psi\right)2}{\left(2 + \frac{p^s_{G1}}{D_{G3}}\right)}. \tag{A.6}$$

(iv) When the endowments are the same across all agents, there exists an equilibrium such that

$$\gamma^{\mathcal{A}} = 0. \tag{A.7}$$

A.1 Exit and Boycott equilibria

Note that we restrict q to be 0. We prove Propositions 4.2–4.4 in this subsection. Lemma A.9 characterizes the conditions where Exit and Boycott equilibria are equivalent to the Benchmark equilibrium when the size of activists is small.
Next, we move to the opposite case where the fraction of activists is sufficiently large such that the brown firm manager will adopt the green technology at $t = 1$. In particular, it is important to investigate state S_{1N} on the off-equilibrium path. Lemma A.10 characterizes the equilibria under Exit. Lemma A.14 analyzes it under Boycott. Lemma A.16 compares the two equilibria in Lemmas A.10 and A.14. Finally, built on the results in which the size of activists is either sufficiently small or sufficiently large, we prove Propositions 4.2 and 4.3 as well as Proposition 4.4.

The following lemma establishes the conditions whether the equilibrium share prices in the Benchmark equilibrium are the same as those in the Exit or Boycott equilibria.

Lemma A.9. *Even in the presence of activist agents, if k_1 is small enough to satisfy the conditions given below, the resulting prices on the equilibrium path under Exit and Boycott are the same as the prices in the Benchmark equilibrium.*

$$\text{Exit: } k_1 \left(D_{G3} + D_{B3} + \gamma^{\mathcal{A}}\right) < D_{G3} + k_1 \frac{\tau}{1+\tau} D_{B3} \text{ and} \tag{A.8}$$

$$k_1 \left(D_{G3} + D_{B3} + \gamma^{\mathcal{A}}\right) < D_{G3} + k_1 \frac{\tau}{1+\tau} D_{B3} + k_1 \frac{D_{G3}}{p^s_{G2}} \left(\psi^{\mathcal{A}} - \psi - \gamma^{\mathcal{A}}\right) \tag{A.9}$$

$$\text{Boycott: } k_1 \left(D_{G3} + D_{B3} + \gamma^{\mathcal{A}}\right) < D_{G3} \tag{A.10}$$

where $\gamma^{\mathcal{A}}$ is given in Lemma A.8.

Next, we move to the case where the fraction of activist agents is sufficiently large and examine the off-equilibrium state S_{1N} and S_{21N}. For the rest of proofs, we consider only equal endowments, $\psi_i = \psi$ and $\theta_{iG} = \theta_{iB} = 1$. Our strategy is as follows. First, we assume that the green technology is adopted and examine the price of brown shares on the off-equilibrium path. Then, we show that the green technology is indeed adopted because the price of brown shares on the off-equilibrium path is too low when the green technology is not adopted.

Lemma A.10. *If the Exit constraint binds at S_{1N}, the Exit constraint binds at S_{21N} and S_{22N}.*

Proof Assume that the Exit constraint does not bind at S_{21N} or S_{22N}. Then, the shares are fairly priced and hence, it holds that

$$p^s_{B2} = p^s_{G2} \frac{D_{B3}}{D_{G3}\,(1+\tau)},$$

and the consumption process of the activist will be

$$\left(c^{\mathcal{A}}_2, c^{\mathcal{A}}_3\right) = \left(\psi, D_{3G} + D_{3G}\right) + \gamma^{\mathcal{A}}_2 \,(1, 1)\,.$$

Note that $\gamma_2^{\mathcal{A}}$ can be pinned down to match the budget at $t = 2$:

$$\psi_2 + p_{G2}^s + p_{B2}^s (1+\tau) + \gamma_2^{\mathcal{A}} \left(1 + \frac{p_{G2}^s}{D_{G3}}\right) = \frac{1}{k_1} p_{G2}^s + \psi_2 + p_{B2}^s \tau,$$

which implies

$$\gamma_2^{\mathcal{A}} = \frac{\left(\frac{1}{k_1} - 1\right) p_{G2}^s - p_{B2}^s}{1 + \frac{p_{G2}^s}{D_{G3}}} = p_{G2}^s \frac{\left(\frac{1}{k_1} - 1\right) - \frac{D_{B3}}{D_{G3}(1+\tau)}}{1 + \frac{p_{G2}^s}{D_{G3}}} < 0,$$

where the last equality is from $\left(1 + \frac{D_{B3}}{D_{G3}(1+\tau)}\right) k_1 > 1$. Then, the aggregate demand for the green shares by the activists is

$$\begin{aligned} & k_1 \cdot \frac{1}{p_{G2}^s} \left(\frac{1}{k_1} p_{G2}^s + \psi_2 - c_2^{\mathcal{A}}\right) \\ =& k_1 \cdot \frac{1}{p_{G2}^s} \left(\frac{1}{k_1} p_{G2}^s - \gamma_{21}^{\mathcal{A}}\right) = 1 - k_1 \cdot \frac{1}{p_{G2}^s} \gamma_{21}^{\mathcal{A}} > 1, \end{aligned}$$

which implies that the aggregate demand of activists and reformers will exceed 1 at S_{22N}. Hence, the Exit constraint binds at S_{21N} and S_{22N}. This completes the proof of the lemma. □

Lemma A.11. *Assume that the green technology is adopted at S_1 under Exit. Then, the followings hold on S_{1N} and S_{21N}:*
(i) The prices of p_{Gt}^s and p_{Bt}^s and the consumption of $c_t^{\mathcal{A}}$ and $c_t^{\mathcal{N}}$ are determined by the following relations:

$$p_{Gt}^s = M_{\texttt{NAD}} \left(c_t^{\mathcal{A}}, \frac{D_{G3}}{k_1} + \frac{\tau}{1+\tau} D_{B3}\right) D_{G3} \tag{A.11}$$

$$p_{Bt}^s = M_{\texttt{NAD}} \left(c_t^{\mathcal{N}}, \frac{D_{B3}}{1-k_1} \frac{1}{1+\tau} + \frac{\tau}{1+\tau} D_{B3}\right) \frac{D_{B3}}{1+\tau} \tag{A.12}$$

$$0 = \left(p_{Bt}^s \frac{1}{1-k_1} + c_t^{\mathcal{N}}\right) - \left(p_{Gt}^s + p_{Bt}^s + \psi\right) \tag{A.13}$$

$$0 = \left(k_1 c_t^{\mathcal{A}} + (1-k_1) c_t^{\mathcal{N}}\right) - \psi, \tag{A.14}$$

where $M_{\texttt{NAD}}$ is given by (A.2),
(ii) As $k_1 \to 1$, $p_{Bt}^s \to 0$,
(iii) $\frac{dp_{Bt}^s}{dk_1} < 0$.

Lemma A.12. *Assume that the green technology is adopted at S_1 under Exit. Then, there exists $\bar{\bar{k}}_{Exit}$ such that if $k_1 > \bar{\bar{k}}_{Exit}$, $p_B^s(S_{1N}) < p_B^s(S_{1A})$ and $p_B^s(S_{21N}) < p_B^s(S_{21A})$.*

Proof Note that $p_B^s(S_{1A}) = p_B^s(S_{21A})$ are constants in the equilibrium where the green technology is adopted. From Lemma A.11(ii) and (iii), there exist $\bar{\bar{k}}_{Exit}$ such that $p_{B1}^s(S_{1N}) < p_{B1}^s(S_{1A})$ and $p_{B1}^s(S_{21N}) < p_{B1}^s(S_{21A})$ if $k > \bar{\bar{k}}_{Exit}$. This completes the proof of the lemma. □

The next lemma highlights that a bigger size of interim activists may hurt in achieving their goal of green technology adoption for Exit.

Lemma A.13. *Assume that the Exit constraint binds at S_{1N} and $k_1 < \bar{\bar{k}}_{Exit}$, where $\bar{\bar{k}}_{Exit}$ is given by Lemma A.9. Set $\underline{\bar{\bar{k}}}_{Exit}$ such that $p^s_{B2}(S_{22N}) = p^s_{B2}(S_{22A})$ when $k_2 = \underline{\bar{\bar{k}}}_{Exit}$. Then, it holds that $\frac{d\underline{\bar{\bar{k}}}_{Exit}}{dk_1} > 0$.*

Proof From Lemma A.10, the Exit constraint also binds at S_{22N}. We combine activists and reformers and denote the aggregate agent as $\mathcal{AR}$. Then, the equilibrium prices of p^s_{G2} and p^s_{B2} are determined by the following system of equations:

$$p^s_{G2} = M_{\texttt{NAD}}\left(c_2^{\mathcal{AR}}, \frac{D_{G3}}{k_2} + \frac{\tau}{1+\tau}D_{B3}\right)D_{G3} \tag{A.15}$$

$$p^s_{B2} = M_{\texttt{NAD}}\left(c_2^{\mathcal{N}}, \frac{D_{B3}}{1-k_2}\frac{1}{1+\tau} + \frac{\tau}{1+\tau}D_{B3}\right)\frac{D_{B3}}{1+\tau} \tag{A.16}$$

$$0 = \left(p^s_{B2}\frac{1}{1-k_2} + c_2^{\mathcal{N}}\right) - \left(\frac{1-k_2}{1-k_1}p^s_{B2} + \psi_1\right) \tag{A.17}$$

$$0 = \left(k_2 c_2^{\mathcal{AR}} + (1-k_2)c_2^{\mathcal{N}}\right) - \psi_1, \tag{A.18}$$

where $c_2^{\mathcal{AR}} = \frac{1}{k_2}\left(k_1 c_2^{\mathcal{A}} + (k_2 - k_1)\, c_2^{\mathcal{R}}\right)$. From (A.16), we have that

$$dp^s_{B2} = \frac{dp^s_{B2}}{dk_1}dk_1 + \frac{dp^s_{B2}}{dk_2}dk_2, \tag{A.19}$$

where

$$\frac{dp^s_{B2}}{dk_1} = \frac{\partial p^s_{B2}}{\partial c_2^{\mathcal{N}}}\frac{dc_2^{\mathcal{N}}}{dk_1}. \tag{A.20}$$

We check the sign of $\frac{dp^s_{B2}}{dk_1}$ first. Taking a derivative of (A.17) with respect to k_1, we have

$$\left(\frac{dp^s_{B2}}{dk_1}\frac{1}{1-k_2} + \frac{dc_2^{\mathcal{N}}}{dk_1}\right) - \frac{1-k_2}{(1-k_1)^2}p^s_{B2} - \frac{1-k_2}{1-k_1}\frac{dp^s_{B2}}{dk_1} = 0,$$

which in turn gives

$$\left(\frac{dp^s_{B2}}{dk_1}\left(\frac{1}{1-k_2} - \frac{1-k_2}{1-k_1}\right) + \frac{dc_2^{\mathcal{N}}}{dk_1}\right) = \frac{1-k_2}{(1-k_1)^2}p^s_{B2},$$

which in conjunction with (A.20), yields

$$\left(\left(\frac{1}{1-k_2} - \frac{1-k_2}{1-k_1}\right) + \left(\frac{\partial p^s_{B2}}{\partial c_2^{\mathcal{N}}}\right)^{-1}\right)\frac{dp^s_{B2}}{dk_1} = \frac{1-k_2}{(1-k_1)^2}p^s_{B2}.$$

Hence, we have that

$$\frac{dp^s_{B2}}{dk_1} > 0. \tag{A.21}$$

Next, we check the sign of $\frac{dp^s_{B2}}{dk_2}$. Taking a derivative of the system of equations (A.16) and (A.17) with respect to k_2, we have that

$$\frac{dp^s_{B2}}{dk_2} = p^s_{B2}\left(A\frac{dc^{\mathcal{N}}_2}{dk_2} - A\frac{D_{B3}}{(1-k_2)^2}\frac{1}{1+\tau}\right) \tag{A.22}$$

$$0 = \left(\frac{dp^s_{B2}}{dk_2}\cdot\frac{1}{1-k_2} + p^s_{B2}\frac{1}{(1-k_2)^2} + \frac{dc^{\mathcal{N}}_2}{dk_2}\right) - \left(\frac{-1}{1-k_1}p^s_{B2} + \frac{1-k_2}{1-k_1}\frac{dp^s_{B2}}{dk_2}\right). \tag{A.23}$$

We can rewrite (A.23) as

$$0 = \left(\frac{dp^s_{B2}}{dk_2}\cdot\frac{1}{1-k_2} + p^s_{B2}\frac{1}{(1-k_2)^2} + \frac{dc^{\mathcal{N}}_2}{dk_2}\right) - \left(\frac{-1}{1-k_1}p^s_{B2} + \frac{1-k_2}{1-k_1}\frac{dp^s_{B2}}{dk_2}\right)$$

$$0 = \frac{dp^s_{B2}}{dk_2}\left(\frac{1}{1-k_2} - \frac{1-k_2}{1-k_1}\right) + p^s_{B2}\left(\frac{1}{(1-k_2)^2} + \frac{-1}{1-k_1}\right) + \frac{dc^{\mathcal{N}}_2}{dk_2},$$

which in conjunction with (A.22) gives

$$-\frac{D_{B3}}{(1-k_2)^2}\frac{1}{1+\tau} = \frac{dp^s_{B2}}{dk_2}\left(\frac{1}{1-k_2} - \frac{1-k_2}{1-k_1}\right) + p^s_{B2}\left(\frac{1}{(1-k_2)^2} - \frac{1}{1-k_1}\right) + \left(\frac{1}{p^s_{B2}A}\frac{dp^s_{B2}}{dk_2}\right),$$

which yields

$$\frac{dp^s_{B2}}{dk_2}\left(\frac{1}{1-k_2} - \frac{1-k_2}{1-k_1} + \frac{1}{p^s_{B2}A}\right) = -\frac{D_{B3}}{(1-k_2)^2}\frac{1}{1+\tau} - p^s_{B2}\left(\frac{1}{(1-k_2)^2} - \frac{1}{1-k_1}\right).$$

Hence,

$$\frac{dp^s_{B2}}{dk_2} < 0. \tag{A.24}$$

Lastly, note that $p^s_{B2}(S_{22N}) = p^s_{B2}(S_{22A})$ when $k_2 = \bar{\bar{k}}_{Exit}$ and that $p^s_{B2}(S_{22A})$ is a constant. With the given signs of (A.21) and (A.24), we have $\frac{d\bar{\bar{k}}_{Exit}}{dk_1} > 0$ so that $dp^s_{B2} = 0$ in (A.19). This completes the proof of the Lemma. □

The effect of τ on the p^s_{B2}

Next, we move to the case of Boycott.

Lemma A.14. *Assume that the green technology is adopted at S_1 under Boycott. Then, the followings hold on S_{1N} and S_{21N}:*
(i) The prices of p^s_{Gt} and p^s_{Bt} and the consumption of $c^{\mathcal{A}}_t$ and $c^{\mathcal{N}}_t$ are determined by the

following relations:

$$p^s_{Gt} = M_{\mathtt{NAD}}\left(c^{\mathcal{A}}_t, \frac{D_{G3}}{k_1}\right) D_{G3} \tag{A.25}$$

$$p^s_{Bt} = M_{\mathtt{NAD}}\left(c^{\mathcal{N}}_t, \frac{D_{B3}}{1-k_1}\right) \frac{D_{B3}}{1+\tau} \tag{A.26}$$

$$\frac{D_{G3}}{p^s_{Gt}} = \frac{D_{B3}p^c_{B3}}{p^s_{Bt}} \tag{A.27}$$

$$\frac{D_{B3}p^c_{B3}\,(1+\tau)}{1-k_1} = \frac{D_{G3}}{p^s_{Gt}}\left(p^s_{Gt} + p^s_{Bt} + \psi - c^{\mathcal{N}}_t\right) + \tau D_{B3}p^c_{B3} \tag{A.28}$$

$$0 = \left(k_1 c^{\mathcal{A}}_t + (1-k_1)c^{\mathcal{N}}_t\right) - \psi, \tag{A.29}$$

where $M_{\mathtt{NAD}}$ is given by (A.2).
(ii) As $k_1 \to 1$, $p^s_{Bt} \to 0$,
(iii) $\frac{dp^s_{Bt}}{dk_1} < 0$.

Lemma A.15. *Assume that the green technology is adopted at S_1 under Boycott. Then, there exists $\bar{\bar{k}}_{Boycott}$ such that if $k_1 > \bar{\bar{k}}_{Boycott}$, $p^s_B\left(S_{1N}\right) < p^s_B\left(S_{1A}\right)$ and $p^s_B\left(S_{21N}\right) < p^s_B\left(S_{21A}\right)$.*

Proof Note that $p^s_B\left(S_{1A}\right) = p^s_B\left(S_{21A}\right)$ are constants in the equilibrium where the green technology is adopted. From Lemma A.14(ii) and (iii), there exist $\bar{\bar{k}}_{Boycott}$ such that $p^s_{B1}\left(S_{1N}\right) < p^s_{B1}\left(S_{1A}\right)$ and $p^s_{B1}\left(S_{21N}\right) < p^s_{B1}\left(S_{21A}\right)$ if $k > \bar{\bar{k}}_{Boycott}$. This complete the proof of the lemma. □

Lemma A.16. *Assume that the green technology is adopted in state S_1 under Exit as well as Boycott. Then, in states S_{1N} and S_{21N}, the brown firm price under the Exit equilibrium, p^s_{Bt}, is larger than that under the Boycott equilibrium.*

Based on the lemmas above, we prove Propositions 4.2, 4.3, and 4.4.

Proof of Proposition 4.2 Recall that we consider the case of homogeneous endowments. We set $\bar{k}_{Exit}$ such that

$$\bar{k}_{Exit}\left(D_{G3} + D_{B3}\right) = D_{G3} + \bar{k}_{Exit}\frac{\tau}{1+\tau}D_{B3}.$$

Then, from Lemma A.9, if $k_1 < \bar{k}_{Exit}$, share prices will be the same as in the Benchmark equilibrium, and the green technology will not be adopted.

Also, we set $\bar{\bar{k}}_{Exit}$ as suggested in Lemma A.12. Then, from Lemma A.12, if $k_1 > \bar{\bar{k}}_{Exit}$, the green technology will not be adopted at $t = 1$. This completes the proof of the proposition. □

Proof of Proposition 4.3 Recall that we consider the case of homogeneous endowments. We set $\bar{k}_{Boycott}$ such that

$$\bar{k}_{Boycott}\left(D_{G3} + D_{B3}\right) = D_{G3}.$$

Then, from Lemma A.9, if $k_1 < \bar{k}_{Boycott}$, share prices will be the same as in the Benchmark equilibrium, and the green technology will not be adopted.

Also, we set $\bar{\bar{k}}_{Boycott}$ as suggested in Lemma A.15. Then, from Lemma A.15, if $k_1 > \bar{\bar{k}}_{Boycott}$, the green technology will not be adopted at $t = 1$. This completes the proof of the proposition. □

Proof of Proposition 4.4 Note that there is no difference between Exit and Boycott in the state S_{1A}. Then, given the technology adoption rule (3.5), the Proposition directly follows from Lemma A.16. □

A.2 Voice Equilibrium

Finally, we move to Voice equilibrium. Lemma A.17 establishes the condition on whether the equilibrium share prices and consumption allocations are the same as those in a Voice equilibrium. Then, the proofs for Propositions 4.5 and 4.6 follow.

Lemma A.17. *The Voice equilibrium consumption allocations are the same as the Benchmark equilibrium consumption allocations when the fraction of activists, k_1, is small, satisfying the following inequality.*

$$k_1 \left(\frac{D_{G3} (1 + \tau)}{D_{B3}} + 1 \right) < \frac{1}{2}. \tag{A.30}$$

Using the lemma above and all the results so far, we prove Propositions 4.5 and 4.6.

Proof of Proposition 4.5 Set $\bar{\bar{k}}_{Voice}$ such that

$$\bar{\bar{k}}_{Voice} \left(\frac{D_{G3} (1 + \tau)}{D_{B3}} + 1 \right) = \frac{1}{2}.$$

Then, from Lemma A.17, the green technology is not adopted. Next consider $k_1 > \bar{\bar{k}}_{Voice}$ and the green technology is adopted.

We conjecture $\hat{k}_{Voice}$ such that

$$\hat{k}_{Voice} \left(\frac{D_{G3}}{D_{B3} (1 - \eta)} + 1 \right) = 1 \tag{A.31}$$

and verify the conjecture below. Assume the fair pricing equilibrium with the green technology. Then, the optimal consumption process is the endowment process and shares are fairly priced. Also, the aggregate demand of activists for the brown shares is given as follows

$$\int_0^{k_1} \theta_{iBt} di = \left(\int_0^{k_1} \theta_{iGt-1} di \right) \frac{p^s_{Gt}}{p^s_{Bt}} + \left(\int_0^{k_1} \theta_{iBt-1} di \right) = \left(\int_0^{k_1} \theta_{iGt-1} di \right) \frac{D_{G3}}{D_{B3} (1 - \eta)} + \left(\int_0^{k_1} \theta_{iBt-1} di \right).$$

Set $t = 1$. Then, the RHS of the above is $k_1 \left(\frac{D_{G3}}{D_{B3}(1-\eta)} + 1 \right)$. Hence, from (A.31), the aggregate demand is less than 1 at $t = 1$ if $k_1 < \hat{k}_{Voice}$. Next, move to $t = 2$. Then, because $\left(\int_0^{k_1} \theta_{iB1} di\right) < 1$ and Voice constraint, $\left(\int_0^{k_1} \theta_{iB2} di\right) < 2$ or the aggregate demand is less than 1 at $t = 2$. This completes the proof of the proposition. □

Proof of Proposition 4.6 The first claim follows from Lemma A.9. The second claim follows by reversing the inequality sign. This completes the proof of the proposition. □

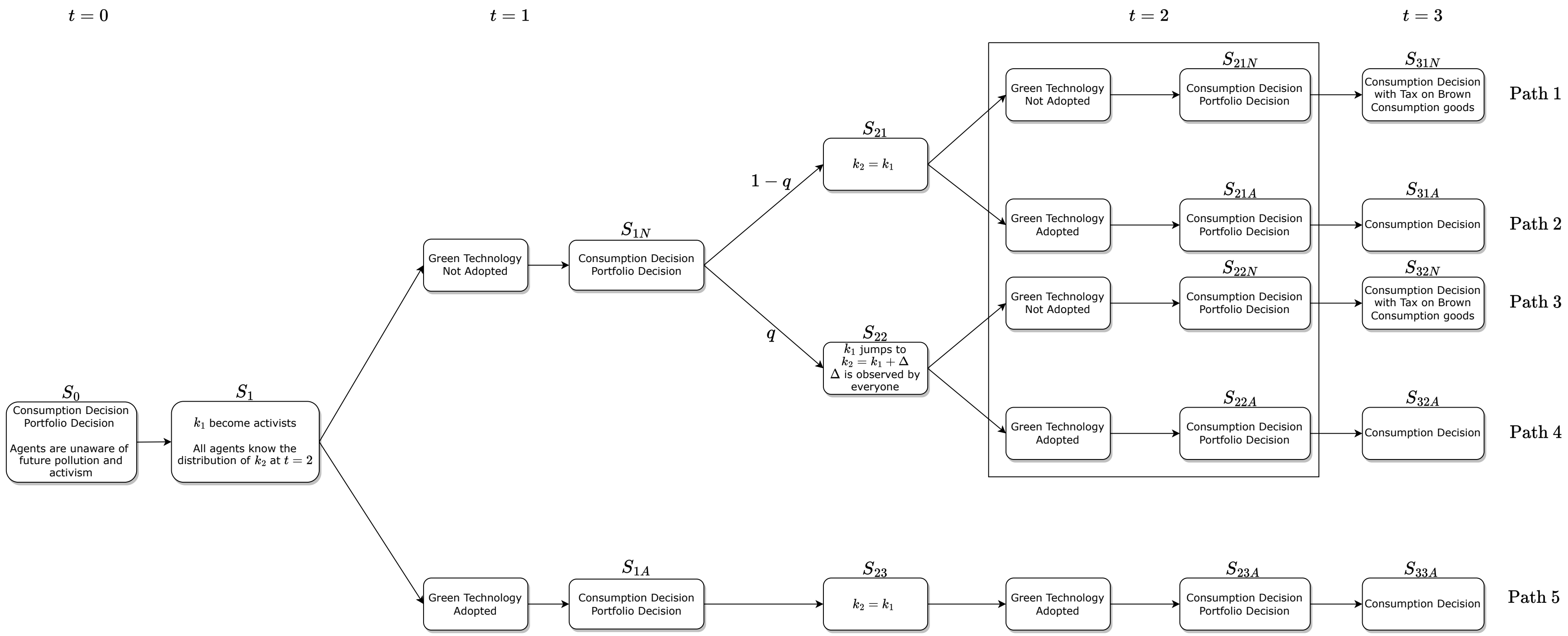


Figure 10: Timeline of the model. The symbols S_0–S_{33A} represent the "state" of the economy. N indicates that firm B's manager does not adopt the green technology while A indicates that the manager adopts. Paths 1–5 denote possible equilibrium paths. In equilibrium, only one path will ex post be realized.